Long run risk is the worst-case scenario: ambiguity aversion and non-parametric estimation of the endowment process

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Introduction

- We do not know the consumption process
  - E.g. debate over nature (and even existence) of long-run risk
- If economists don't, perhaps consumers don't either
- This paper:
  - Assume Epstein–Zin preferences
  - What is the worst-case plausible process for consumption?
  - What happens if investors price using that model?

Introduction

- What does model uncertainty mean?
  - Parametric uncertainty: unsure about the parameters of a known process (e.g. an AR(1))
  - Full model uncertainty: you don't even know the order of the ARMA representation
    - Infinite-dimensional problem

Introduction

- We allow for full model uncertainty
- The big idea: the worst-case model is literally the long-run risk model (small, persistent trend in growth)
  - Very painful
  - Very difficult to reject
  - Gives a rigorous justification for LRR models (not just moment matching)
- Implications:
  - Very high risk premia for small risk aversion
  - Matches many asset pricing moments
  - Asset prices are strongly procyclical
  - No arguments about the true endowment process

Consumption Processes

- The true data generating process is

$$\begin{aligned} \Delta c_t &= \bar{b}(L)\,\varepsilon_t \\ \varepsilon_t &\sim h_\varepsilon = N(0,1) \end{aligned}$$

where Δc is log consumption growth, L is the lag operator

$$\bar{b}(L)\,\varepsilon_t = \sum_{j=0}^{\infty} \bar{b}_j \varepsilon_{t-j}$$

- Only restriction on $\bar{b}$ is that Δc is stationary, spectrum is finite, non-zero

Preferences

Given a known model, agents have Epstein–Zin preferences, unit EIS

$$v_t = (1-\beta)\, c_t + \frac{\beta}{1-\alpha} \log E_t \exp\left(v_{t+1}\,(1-\alpha)\right)$$

- α is the coefficient of relative risk aversion
- β time discount factor
- Unit EIS leads to analytic solutions

- Epstein–Zin preferences with a known model

$$vc\left(\Delta c^{t};b\right)=\frac{\beta}{1-\beta}\frac{1-\alpha}{2}b\left(\beta\right)^{2}+\sum_{j=1}^{\infty}\beta^{j}E_{t}\left[\Delta c_{t+j}|b\right]$$

- **Key result:** agents averse to long-run risk
    - $b\left(\beta\right)$ measures the total discounted effect of ε on consumption
    - If $\alpha=1$, reduces to power utility, dynamics irrelevant

Ambiguity

- The agent possesses a subjective distribution over b (informed by estimation)
    - Standard Bayesian analysis is (maybe) possible but intractable
- She does not reduce her uncertainty over b into a unique distribution over Δc_t sequences
- Instead, she identifies a "worst case" data generating process
- Captures key feature of risk-averse behavior: we overweight bad outcomes

Preferences - Robustness over Propagation

- Find the worst-case model of propagation, b^w

$$
\begin{aligned}
b^w &= \arg\min_b E\left[vc\left(\Delta c^t; b\right)|b\right] + \lambda g\left(b\right) \\
&= \arg\min_b \frac{\beta}{1-\beta}\frac{1-\alpha}{2} b\left(\beta\right)^2 \sigma_\varepsilon^2 + \lambda g\left(b\right)
\end{aligned}
$$

  - $g\left(b\right)$ measures the plausibility of a model
  - λ and α closely related (directly linked in the paper)
- Note the unconditional expectation
  - Agent chooses a worst-case model once and for all
  - Preferences are not fully dynamic

Deriving $g(b)$

- $g(b)$ encodes the agent's subjective beliefs regarding the likelihood of a given b.
- Desired properties of $g(b)$
  1. Results from a realistic estimation procedure
  2. Imposes minimal structure on the class of b considered

$\Rightarrow$ non-parametric spectral estimation

Non-parametric Spectral Estimation

- Agent estmates a finite-order *AR* (or *MA*) with the lag order increasing with sample size
    - Berk (1974), Brockwell and Davis (1988)
    - Asymptotically non-parametric
- The confidence set contains models with minimally constrained dynamics
    - Parametric point estimate; non-parametric confidence interval
- The models will *generically* be outside the parametric class of the *AR* or *MA* estimated from any given sample

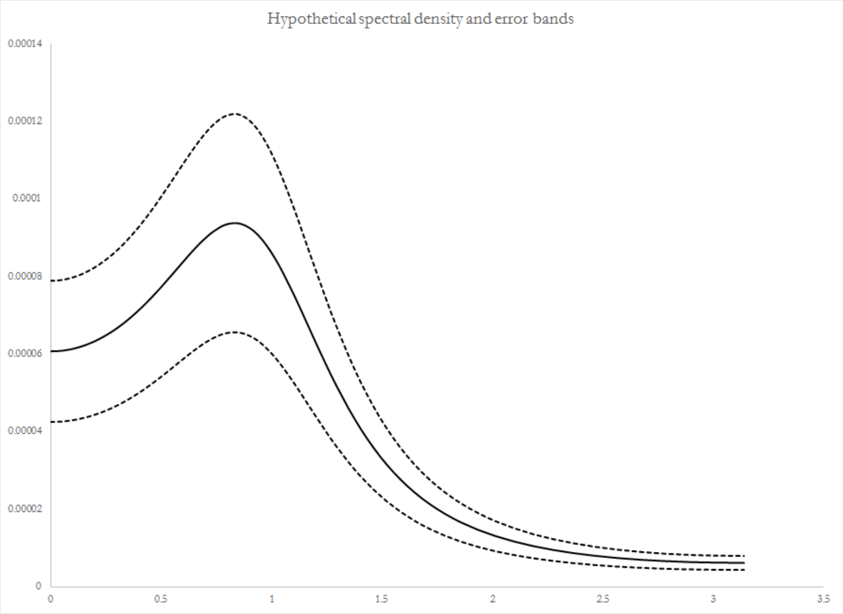
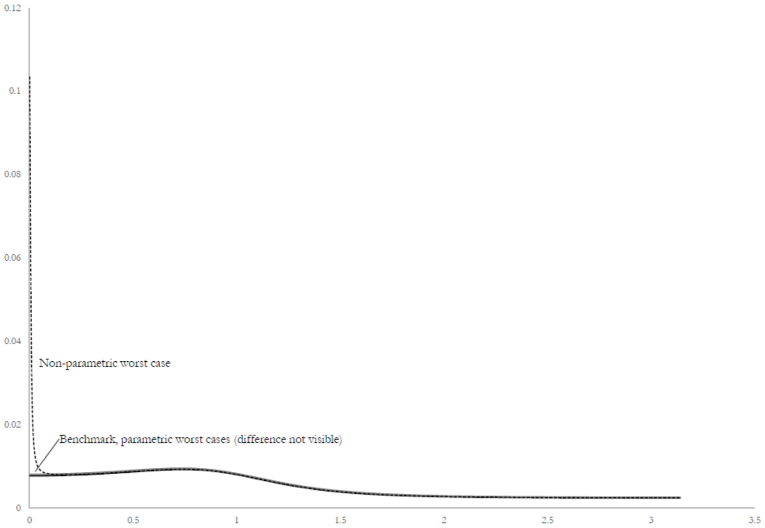

Hypothetical spectral density and error bands
0.00014
0.00012
0.0001
0.00008
0.00006
0.00004
0.00002
0
0
0.5
1
1.5
2
2.5
3
3.5

Figure 6. Benchmark and worst-case transfer function



Simple benchmark: white noise

- True model:

$$\Delta c_t = \bar{b}_0 \varepsilon_t$$

- Worst-case model

$$\begin{aligned} b_j^w &= \varphi \beta^j \text{ for } j > 0 \\ b_0^w &= \bar{b}_0 + \varphi \end{aligned}$$

- Exactly equivalent to

$$\begin{aligned} \Delta c_t &= x_t + \mu_t \\ x_t &= \beta x_{t-1} + v_t \end{aligned}$$

- μ_t and v_t independent white noise

Simple benchmark: white noise

- If consumption growth is white noise (and the agent estimates that) then the worst-case model is *literally* the long-run risk model
  - The agent fears a persistent trend in consumption growth
- The persistence of the trend is exactly the time discount factor
  - Match the deviation from the model to the impact on utility
  - Less persistent $\Rightarrow$ easier to detect
  - More persistent $\Rightarrow$ placing deviations on dates you don't care about (e.g. fractional integration)
- This is an *extremely* persistent trend
  - Half-life of 70 years
  - Bansal and Yaron (2004): half life = 3 years

Asset pricing and calibration

- Examine returns on levered consumption claims
  - Standard model of equity
- Moments to match:
  - Hansen–Jagannathan bound
  - Equity premium and Sharpe ratio
  - Volatility of equity prices
  - Predictability of returns
  - Interest rate behavior

Calibration

Parameter	Value	Description
β	$0.99^{1/4}$	Time preference 1% per year
γ	5.62	Equity leverage $(D_t = C_t^{\gamma})$
λ	0.116	For H-J bound of 0.33 per year
α	5.31	Implied by choice of λ
$std\,(\Delta c)$ (annual)	2%	Annual consumption vol.

Asset pricing moments

Moment	Model	Standard EZ	Data
HJ bound	0.33	0.09	–
$E\left[r_m - r_f\right]$	7.00	2.09	7
$std\left(r_m\right)$	19.69	11.24	21
$std\left(E_t\left[r_{m,t+1} - r_{f,t+1}\right]\right)$	3.06	0	–
$AC1(P/D)$	0.94	N/A	0.91
$std\left(P/D\right)$	0.24	0	0.40

Properties of the worst-case model

- The worst-case model is an ARMA(1,1) with AR root of β
- Can the agent reject the worst-case model?
- Generate data under the benchmark; what is the probability the agent rejects the worst-case at the 5% level?

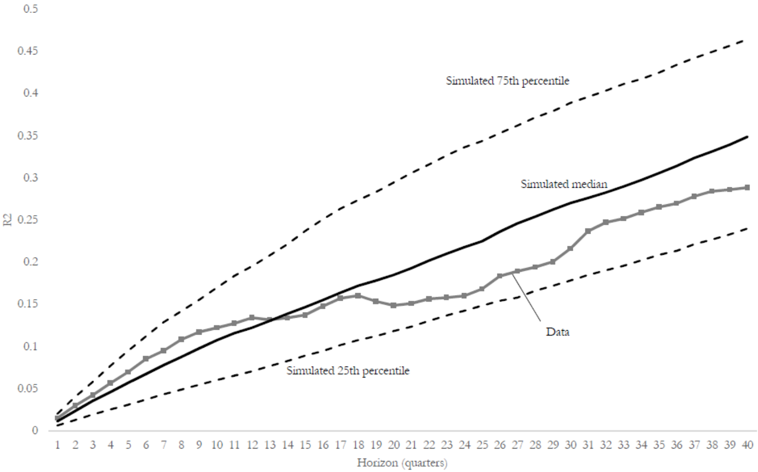
Test	50-year sample	100-year sample
Ljung–Box	4.6%	4.9%
ARMA(1,1) LR	5.4%	6.3%
Newey–West	9.2%	9.5%

- The agent is extremely unlikely to reject the worst-case model
  - Even for a correctly specified likelihood ratio test!

Predictability

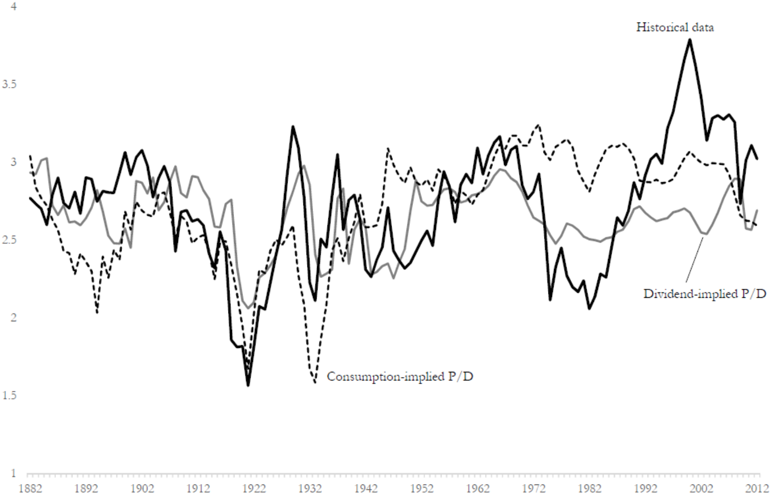
- Under the pricing model, consumption growth is persistent
- So agents act like good times will continue
    - Looks like naive extrapolation
    - Or 'natural expectations' (Fuster et al., 2011)
- Makes asset prices procyclial and returns predictable

Figure 3. Empirical and model-implied R2's from return forecasting regressions



Notes: Black lines give results from simulated regressions on 50-year samples. The grey line plots R2s from regressions of aggregate equity returns on the price/dividend ratio in the post-war sample.

Figure 4. Historical and model-implied log price/dividend ratios



Notes: The historical price/dividend ratio is for the S&P 500 from Robert Shiller. The consumption-based measure uses data from Barro and Ursua (2008). The dividend-based measure uses Shiller's data on dividends. All three series have the same mean by construction.

Summary

- We study asset pricing with full model uncertainty
- The pricing model displays strong persistence
- Induces a large and volatile equity premium
- More generally: shows what kind of uncertainty models need to allow
    - Don't just let people be unsure of the AR(1) parameter