Uncertainty shocks as second-moment news shocks

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Abstract

We provide evidence on the relationship between aggregate uncertainty and the macroeconomy. Identifying uncertainty shocks using methods from the news shocks literature, the analysis finds that innovations in realized stock market volatility are robustly followed by contractions, while shocks to forward-looking uncertainty have no significant effect on the economy. Moreover, investors have historically paid large premia to hedge shocks to realized but not implied volatility. A model in which fundamental shocks are skewed left can match those facts. Aggregate volatility matters, but it is the *realization* of volatility, rather than uncertainty about the future, that has been associated with declines.

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1 Introduction

A growing literature in macroeconomics studies the effects of news shocks on the economy. Models with rational forward-looking agents imply that pure changes in expectations about the future – news shocks – can induce a response in the aggregate economy. The existing literature has focused on *first-moment* news shocks: news about the average future path of the economy. For example, the literature on total factor productivity (TFP) and real business cycles identifies two types of TFP shocks: surprise innovations in TFP, and news about the future level of TFP that has no effects on TFP on impact.[1]

This paper extends the estimation to *second-moment* news shocks. Whereas the work described so far studies changes in the expected future growth rates, we study changes in expected future *squared* growth rates. News about the expectation of future squared growth rates represents a change in the conditional variance – that is, it is an *uncertainty shock* (Beaudry and Portier (2014)).

Our goal is to use a news shock identification scheme to study the effects of uncertainty on the real economy. How uncertainty affects the economy is objectively interesting given that it clearly varies over time, often as a first-order consequence of policy choices. Moreover, a growing literature has developed a range of theoretical channels through which uncertainty shocks might cause recessions, though in most of those models the sign of the effect of uncertainty shocks is ambiguous.[2] Our analysis directly tests those theories by quantifying how the economy responds to identified shocks to uncertainty.

In order to identify pure uncertainty shocks, we distinguish between current squared growth rates and news about future squared growth rates, i.e. between $(\Delta TFP_t)^2$ and $E_t\left[(\Delta TFP_{t+1})^2\right]$. For reasons discussed below, we measure second moments using aggregate stock returns instead of TFP. So our uncertainty shock is an increase in the variance of the conditional distribution of future stock prices, while the analog to the first-moment impact shock is the surprise in the size of squared changes in stock prices – realized volatility – during the current period.

The key distinction between the two shocks is that realized volatility is not the same as uncertainty about the future. Models of the effects of uncertainty, such as those with wait-and-see or precautionary saving effects, are driven by variation in agents' subjective distributions of future shocks, as opposed to the variance of the distribution from which today's shocks were drawn. The importance of that distinction is part of the basic message of this paper.

The analysis focuses on the effects of uncertainty about the aggregate stock market. Our concept of uncertainty therefore refers to the aggregate value of the largest firms in the US economy. Stock returns are a useful indicator because they should be affected by any shock that affects the value of firms. They have three further advantages: the availability of high-frequency data allows accurate

[1] Beaudry and Portier (2006), Barsky and Sims (2011), and Barsky, Basu, and Lee (2015)

[2] For theoretical models, among many others, see Basu and Bundick (2017), Bloom et al. (2016), and Leduc and Liu (2016). Bloom et al. (2016) discuss how in their model uncertainty shocks can have both expansionary and contractionary effects, and similar forces are present in other settings.

calculation of realized volatility, option prices can be used to provide information about expected volatility, and measures of stock market volatility such as the VIX have been widely used in past research on uncertainty shocks, making it easy to compare our work to the existing literature. Other data sources typically only allow one to calculate either uncertainty (e.g. the surveys studied in Bachmann, Elstner, and Sims (2013)) or realized volatility (e.g. the cross-sectional variance of income growth in Storesletten, Telmer, and Yaron (2007)), but not both.

We identify the two shocks using the identification scheme of Barsky, Basu, and Lee (2015), which identifies a news shock in a VAR as the rotation of the reduced-form shocks that predicts the variance of future stock returns and is also orthogonal to the variance of returns in the current period. In two recent state-of-the-art models of the effects of aggregate uncertainty from Basu and Bundick (2017) and Bloom et al. (2017), this identification scheme accurately uncovers the true structural uncertainty shock.

The paper's main result is that the two shocks have statistically and economically different effects on the economy. Across a range of VAR specifications, increases in contemporaneous realized volatility are associated with declines in output, consumption, investment, and employment. On the other hand, the identified uncertainty shock is estimated to have no significant effect on the real economy, even though it accounts for more than a third of the variation in overall uncertainty and is strongly correlated with declines in stock prices. And the difference between the responses of the economy to the realized and expected volatility shocks is itself statistically significant in our benchmark specification, indicating that the failure to find uncertainty shocks to be contractionary is not simply due to low statistical power.

The uncertainty shocks are also not small. They have statistically significant forecasting power for future stock market volatility at horizons of one to two years (which is typical for stock market volatility and similar to the length of the uncertainty shocks measured by Bloom (2009)), they account for 30–60 percent of the total variance of uncertainty, and we show in regressions that option-implied volatility explains more of the variation in expectations of future volatility than lags of volatility itself do. In other words, option market investors appear to have economically meaningful information about future uncertainty that is not contained in the time series of past realized volatility. It is that information that drives our identification.

The last section of the paper presents a simple model that qualitatively matches the evidence on the effects of realized volatility and uncertainty shocks. Its key mechanism is that shocks to technology are negatively skewed. Negative skewness means that large shocks – which cause high realized volatility – also tend to be negative shocks, immediately generating the observed negative correlation between realized volatility and output. When we estimate the same VAR in the new model that we estimate in the data, we find qualitatively and quantitatively similar results. Moreover, the identified shocks in the simulated VAR for the skewed model are strongly correlated with the simulated structural shocks, providing theoretical support for our identification scheme. So negative skewness yields a simple explanation for our empirical findings.

There are two important further pieces of evidence in favor of the skewness hypothesis. First, changes in a wide variety of measures of real activity are negatively skewed in the data, as are stock returns. Second, investors have historically paid large premia for insurance against high realized volatility and extreme negative stock returns in the last 30 years, whereas the premium paid for protection against increases in *expected* volatility or uncertainty has historically been near zero or even positive. Those findings are consistent with uncertainty having no effects on the economy in equilibrium. We show that the model qualitatively matches both the empirical left skewness and the large premium on realized volatility compared to shocks to volatility expectations.

To be clear, our claim is not that no type of uncertainty could be bad for the economy. There is a very wide range of different measures of uncertainty. We focus on the stock market for the reasons discussed above, but other types of uncertainty certainly might matter, such as policy, interest rate, or idiosyncratic uncertainty.

In addition to the macroeconomic studies discussed above, our work is also closely related to an important strand of research in finance. It has long been understood in the asset pricing literature that expected and realized volatility, while correlated, have important differences (e.g. Andersen, Bollerslev, and Diebold (2007)). A jump in stock prices, such as a crash or the response to a particularly bad macro data announcement, mechanically generates high realized volatility. On the other hand, news about future uncertainty, such as an approaching presidential election, increases expected volatility (Kelly, Pastor, and Veronesi (2016)). Shocks to realized and expected future volatility are correlated, but they are not as strongly correlated as one might expect – in our sample, 60–70 percent. This means that it is possible to identify in the data shocks to expectations that are orthogonal to realizations, and it is well established that they are priced differently by investors (Broadie, Chernov, and Johannes (2007)).

Our work is related to a large empirical literature that studies the relationship between aggregate volatility and the macroeconomy. A wide range of measures of volatility in financial markets and the real economy have been found to be countercyclical.[3] To identify causal effects, a number of papers use VARs, often with recursive identification, to measure the effects of volatility shocks on the economy.[4] Ludvigson, Ma, and Ng (2015), like us, distinguish between different types of uncertainty. They show that variation in uncertainty about macro variables is largely an endogenous response to business cycles, whereas shocks to financial uncertainty cause recessions.[5] Similarly, Caldara et al. (2016) use a penalty-function based identification scheme to distinguish between

[3]Gilchrist, Sim, and Zakrajsek (2014). See, among others, Campbell et al. (2001), Storesletten, Telmer, and Yaron (2004), Guvenen, Ozkan, and Song (2014), Eisfeldt and Rampini (2006), Alexopoulos and Cohen (2009), and Baker, Bloom, and Davis (2015).

[4]See Bloom (2009) and Basu and Bundick (2015), who study the VIX; and Baker, Bloom, and Davis (2015) and Alexopoulos and Cohen (2009), who study news-based measures of uncertainty. Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (2015) measure uncertainty based on squared forecast errors for a large panel of macroeconomic time series (using a two-sided filter to extract a latent volatility factor). Baker and Bloom (2013) use cross-country evidence to argue that there is causal and negative relationship between uncertainty and growth. See also Born and Pfeifer (2014), Caldara et al. (2016), Fernandez-Villaverde et al. (2011) , Fernandez-Villaverde et al. (2013), Leduc and Liu (2016).

[5]Other papers arguing that causality could run from real activity to volatility and uncertainty include Decker, D'erasmo and Boedo (2016), Berger and Vavra (2013), Ilut, Kehrig and Schneider (2015), and Kozlowski, Veldkamp, and Venkateswaran (2016).

the effects of uncertainty and financial conditions. A key distinction between our work and those two papers is that we focus on the distinction between uncertainty expectations and realizations. Moreover, unlike most past work (Ludvigson, Ma, and Ng (2015), Caldara et al. (2016), Carriero, Clark, and Marcellino (2016) and Popiel (2017), excepted), our identification scheme builds on the news shock literature, rather than using a more restrictive purely recursive setup.

2 Preliminary evidence

We begin by briefly presenting simple regressions that illustrate the basic results of the paper. Table 1 reports results of regressions of monthly growth rates of employment and industrial production on the current value and four lags of realized S&P 500 volatility and option-implied volatility. We discuss the data in more detail below, but realized volatility is calculated as the sum of daily squared returns on the S&P 500 index each month and option-implied volatility is nearly identical to the VIX and it is calculated based on option prices at the end of each month. All variables are converted to z-scores to aid interpretation of the coefficients. We report only the average of the coefficients on realized and implied volatility (they are relatively stable across lags).

The first column in the two sections of table 1 shows results from regressions that include only option-implied volatility (V_1). That column shows that high implied volatility – often interpreted as a measure of uncertainty – is associated with declines in employment and industrial production. The second column adds realized volatility (RV) to the regression. Once realized volatility is included, implied volatility no longer carries a negative coefficient – the coefficient is actually *positive* in both regressions. So table 1 provides our simplest piece of evidence that it is *realized* rather than implied volatility that is most clearly associated with declines in real activity. And the effects are economically significant: a unit standard deviation increase in realized volatility is associated with declines in employment and industrial production growth of 0.2 standard deviations.

The evidence in table 1 is very simple: it includes no additional controls, it allows only a very specific and constrained lead/lag relationship, it does not allow us to trace out dynamic responses, and it does not identify structural shocks. The regressions also measure uncertainty purely based on option-implied volatility, even though option prices include risk premia. We therefore focus the majority of our analysis on vector autoregressions, which address those various concerns. But those basic results will continue to hold in the richer specifications.

3 Empirical framework

This section describes how we define, measure, and identify uncertainty shocks in the data.

3.1 Conditional variances

Denote the log of the total return stock index as s_t. Uncertainty about the future value of the stock market relative to its value today is measured in this paper as the conditional variance, $Var_t\left[s_{t+n}\right]$.

The one-period log stock return is $r_t \equiv s_t - s_{t-1}$. If returns are unpredictable and time periods are sufficiently short that $E_t r_{t+1} \approx 0$, we have:

$$Var_t[s_{t+n}] = E_t\left[\sum_{j=1}^{n-1} r_{t+j}^2\right] - \sum_{j=1}^{n-1} E_t[r_{t+j}]^2 \quad (1)$$

$$\approx E_t\left[\sum_{j=1}^{n-1} r_{t+j}^2\right] \quad (2)$$

When returns are unpredictable (which is very nearly true empirically, especially at short horizons), the conditional variance of stock prices on some future date is equivalent to the expected total variance of returns over that same period.[6] Under standard conditions on the returns process, as the length of a time period approaches zero, the second line becomes an equality. Writing the conditional variance in (2) as an expectation directly connects the analysis to the news shock literature, which studies changes in expectations. The result in (2) says that, when stock returns are unpredictable, variation in uncertainty about the future is the same as variation in expectations of volatility in the future – uncertainty and expected volatility are equivalent.

Whereas the literature on news about TFP studies $E_t\left[\sum_{j=1}^{n} \Delta tfp_{t+j}\right]$ where Δtfp is the first difference of log TFP, here we study second-moment expectations: the expectation of future squared returns ($r^2 = (\Delta s)^2$), which is simply the conditional variance of future stock prices. Second-moment news shocks are shifts in expected future squared returns. Based on equation (2), throughout the paper, we refer to second-moment news shocks, or expected volatility shocks, equivalently as uncertainty shocks.

In the literature on TFP news shocks, there is also the contemporaneous innovation in TFP, $tfp_t - E_{t-1}tfp_t$. The analog here is the innovation in *realized volatility,* $r_t^2 - E_{t-1}\left[r_t^2\right]$. The conditional variance of future stock prices, $Var_t[s_{t+n}]$, is equal (when returns are calculated at high frequency) to cumulative expected future realized volatility.

So the analysis parallels the first-moment news shock literature closely. Anywhere past work talks about Δtfp, it is replaced here with r^2, both when looking at realization shocks and at news. First-moment news shocks are changes in the expectation of future values of Δtfp, holding constant the current innovation in Δtfp. Second-moment news shocks are changes in the expectation of future values of r^2, holding constant the current innovation in r^2 (current realized volatility).

An important concern is that since stock prices are forward-looking, an innovation in uncertainty about the future should cause current stock prices to change, meaning that we might expect it to be impossible (or illogical) to construct a shock to uncertainty that is orthogonal to current r^2. But if uncertainty shocks have a linear effect on returns, then they have a quadratic effect on r^2, making

[6] In practice, we work with daily returns where the zero-mean approximation holds strongly, as documented in the literature. In the notation of continuous time models, $E_t\left[r_{t+1}^2\right]$ is $O(\Delta t)$, while $E_t[r_{t+1}]^2$ is $O\left(\Delta t^2\right)$, where Δt is the length of a time period. So as the time period shrinks, the terms involving squared expected returns become negligible.

r^2 in fact generally uncorrelated with uncertainty shocks. We expand on this point below and show that in three benchmark models, structural uncertainty shocks are actually contemporaneously uncorrelated with realized volatility.

One last minor issue is that we have data on daily stock returns, but data on real activity only at the monthly level. We therefore aggregate volatility to a monthly frequency. Specifically, we define realized volatility in month t, RV_t, as

$$RV_t \equiv \sum_{days \in t} r_i^2 \tag{3}$$

We then have

$$Var_t \left[s_{t+n} \right] \approx E_t \left[\sum_{j=1}^{n} RV_{t+j} \right] \tag{4}$$

Again, the approximation is only due to discreteness – if we had truly continuous data instead of sampling only at the daily level, (2) and (4) would hold exactly (for unpredictable returns). Given how small average daily stock returns are (less than 0.05 percent), the approximation errors here are quantitatively unimportant.

We study daily squared returns for a number of reasons. First, the arguments linking stock return volatility to the conditional variance of the level of stock prices rely on time periods being short. Second, monthly returns are a much noisier measure of the actual volatility of returns than are daily returns.[7] Since our goal is to control for current volatility, we want to measure that volatility as well as possible, which is aided by using as many data points as possible – a single observation is an extraordinarily poor way to measure a variance. Furthermore, the VIX index, which has been widely used in past work to measure uncertainty about the future, is motivated theoretically as a measure of expectations of the future volatility of returns in continuous time. All of those factors would actually suggest using data at frequencies even higher than a single day. Such data is not easily accessible over long periods, though, whereas daily data is widely available with a very long time series. Finally, we also study below the behavior of variance swaps, which are derivatives whose payoffs depend explicitly on daily squared returns.

3.2 Vector autoregressions

To identify uncertainty shocks and estimate their effects, we estimate VARs of the form

$$\begin{bmatrix} RV_t \\ Y_t \end{bmatrix} = C + F(L) \begin{bmatrix} RV_{t-1} \\ Y_{t-1} \end{bmatrix} + A\varepsilon_t \tag{5}$$

[7] If returns are Gaussian with a monthly variance of σ_m^2, for example, the monthly squared return is equal to σ_m^2 multiplied by a χ_1^2 random variable (the subscript denoting degrees of freedom), since it is a single squared realization. If there are on average 21 trading days in a month, then the sum of daily squareds return is equal to $\sigma_m^2/21$ (due to returns being serially uncorrelated) multiplied by a χ_{21}^2 random variable. The variance of the sum of daily squared returns is then 21 times smaller than the variance of the monthly return, showing that using high-frequency data gives a much more accurate estimate of the true underlying current volatility.

where RV_t is realized volatility from (3), Y_t is a vector including measures of real activity, variables that help forecast future values of realized volatility, and other controls, C is a vector of constants, and $F(L)$ is a matrix polynomial in the lag operator, L. ε_t is a vector of uncorrelated innovations with unit variances and A is the lower-triangular Cholesky factorization of the variance matrix of the reduced-form innovations. (5) can be estimated by ordinary least squares. The VAR has a moving average (MA) representation,

$$\begin{bmatrix} RV_t \\ Y_t \end{bmatrix} = (I - F(1))^{-1} C + B(L) A\varepsilon_t \tag{6}$$

$$\text{where } B(L) = \sum_{j=0}^{\infty} B_j L^j = (I - F(L))^{-1} \tag{7}$$

The shocks to realized volatility and uncertainty are identified under a timing restriction. The realized volatility shock is treated as moving first, so that its impulse response functions (IRFs) are defined as

$$\frac{\partial E\left[Y_{t+j} \mid RV_t, RV^{t-1}, Y^{t-1}\right]}{\partial RV_t} = B_j A_{(:,1)} \tag{8}$$

where $A_{(:,1)}$ denotes the first column of A. E denotes the expectation operator conditional on the VAR (5) and RV^{t-1} and Y^{t-1} are the histories of RV and Y up to date $t-1$. As in Hamilton (1994), the impulse response is defined as the average change in expectations for the future following a unit innovation in realized volatility. We do not view the realized volatility shock as a structural shock – obviously volatility in the stock market depends on many different underlying shocks. Instead, the IRF tells us *on average* how the economy (output, employment, etc.) changes when realized volatility changes.

The second estimated shock is the residual innovation in uncertainty over the following n periods, $Var_t[s_{t+n}]$ or, equivalently, the residual innovation in expectations of future volatility. It is the component of $E_t \sum_{j=1}^{n} RV_{t+j} - E_{t-1} \sum_{j=1}^{n} RV_{t+j}$ that is orthogonal to $RV_t - E_{t-1}RV_t$. In other words, the uncertainty shock is ordered second, following realized volatility.

The total change in cumulative expected volatility up to time $t+n$ is constructed as

$$E_t \sum_{j=1}^{n} RV_{t+j} - E_{t-1} \sum_{j=1}^{n} RV_{t+j} = \left(e_1 \sum_{j=1}^{n} B_j \right) A\varepsilon_t \tag{9}$$

where $e_1 = [1, 0, ...]$. The parameter n determines the horizon over which the news shock is calculated. Cumulative expected volatility depends on the sum of the first rows of the MA matrices up to lag n. The innovation to expectations over horizon n, i.e. the news about future volatility, is then simply the linear combination of shocks represented by $e_1 \sum_{j=1}^{n} B_j A$. As in Barsky, Basu, and Lee (2014) and Barsky and Sims (2011), we then orthogonalize that linear combination with respect to the innovation to RV_t so that the impact shock to RV_t is uncorrelated with the news shock.[8] The VAR is only partially identified in that it identifies two shocks and is unstructured otherwise.

The uncertainty shock, denoted u_t, is equal to the component of $\left(e_1 \sum_{j=1}^{n} B_j\right) A\varepsilon_t$ that is orthogonal to the RV shock, $e_1 A\varepsilon_t$, and its impulse responses are defined as

$$\partial E\left[Y_{t+j} \mid u_t, RV_t, RV^{t-1}, Y^{t-1}\right] / \partial u_t$$

Again, the VAR yields the average behavior of the economy following a unit increase in uncertainty (conditional on RV_t), as measured by u_t. The measure of uncertainty here thus obviously depends on the variables included in the VAR. Our measure of volatility news is, strictly, the change in expectations of future realized volatility conditional on the vector $[RV_t, Y_t']'$ and the estimated VAR coefficients.

So far the shocks have been defined purely statistically. The next subsection discusses when they have a structural interpretation and shows that the identification holds in recent benchmark models.

An important concern with our identification scheme is that it could be prone to overfitting volatility news. In our most general model, we will have six variables and four lags, meaning that future volatility is being forecast with 24 degrees of freedom. We will thus argue that it is important to restrict the model somewhat to alleviate overfitting and ensure that the results are consistent with economic priors.

The two impulse responses defined above are only identified up to some normalization. In the main analysis, we rescale them so that they have the same effect on uncertainty. The uncertainty/news shock is reported as a unit standard deviation impulse, while the realized volatility shock is rescaled so that its cumulative effect on uncertainty (i.e. its IRF for realized volatility over the next 24 months) is the same as that for the news shock. The shocks then have equal effects on uncertainty about the future and differ only in their effect on realized volatility on impact.

We set $n = 24$ months, which is the horizon over which we examine IRFs (past work finds that volatility shocks have half-lives of 6–12 months, so 24 months represents the point at which the average shock has dissipated by 75 percent or more). A larger value of n reduces power, since long-term effects are relatively difficult to estimate, while a smaller value identifies shocks that may be less relevant for economic decisions. We find similar results for n between 12 and 60 months.

3.3 Identification and the structural models

The identification scheme involves a timing restriction; the uncertainty shock is orthogonalized with respect to realized volatility (it moves second). In what type of model would the timing scheme make sense? The required assumption is that structural uncertainty shocks cannot affect realized volatility contemporaneously. That restriction is perhaps worrisome. Any shock that

[8] The orthogonalized innovation is simply $e_1 \sum_{j=1}^{n} B_j \bar{A}\varepsilon_t$, where $\bar{A}$ is equal to A but with the first column set to zero.

increases uncertainty about the future might be expected to reduce stock prices, and thus create a realization of volatility. But a shock that *decreases* uncertainty also changes stock prices, just in the opposite direction, and thus also creates realized volatility. So, intuitively, since there is a quadratic relationship between shock realizations and realized volatility – both large positive and large negative shocks create high RV – there need not be any nonzero correlation between uncertainty shocks and RV.

As a formal example, if shocks to uncertainty are symmetrically distributed (see Amengual and Xiu (2014) for an analysis of upward and downward jumps in stock market volatility) and they have a linear relationship with stock returns, then they will have exactly zero correlation with squared stock returns. Empirically, monthly changes in the log of S&P 500 implied volatility are close to symmetric. The 10th, 25th, 75th, and 90th percentiles are -0.59, -0.34, 0.25, and 0.62. Deeper into the tails, changes are skewed somewhat to the right (the skewness is 1.21 overall), but the bulk of the distribution is overall not far from being symmetrical, suggesting that the idea that uncertainty shocks should not have a first-order effect on realized volatility is reasonable.

Perhaps more importantly, our identification scheme successfully identifies the uncertainty shocks in two state-of-the-art models of the effects of uncertainty, in addition to the new model that we present below. The first model, from Basu and Bundick (2017; BB from here on), is a new-Keynesian model of the business cycle that features fluctuations in the volatility of innovations to consumption demand. The second model, from Bloom et al. (2017; RUBC from here on), is a real business cycle model that incorporates fixed costs of adjustment in investment, inducing responses to uncertainty at the micro level.[9] The BB and RUBC models capture the two most prominent channels through which uncertainty has been proposed to affect the economy: precautionary savings and wait-and-see effects, respectively.

Finally, the model developed in this paper is a real business cycle model with time-varying uncertainty about the volatility of productivity and also aggregate technology shocks that are skewed left. We therefore refer to it as the RSBC model ("really skewed business cycles", to highlight the key difference with Bloom et al.'s (2017) RUBC – "really uncertain business cycles").

As an experiment, we estimate our VAR specification in simulations of the three models. The three models are all specified in discrete time with time periods equal to a quarter in the case of BB and RUBC and a month in the case of RSBC, so there is not an exact mapping in the models to the daily realized volatility that we study empirically. We approximate it here as the square of the total realized return in the model in each period (after subtracting mean returns as a normalization). The table below reports the correlation of the identified uncertainty shock with the true shock to uncertainty in the three models:

Correlation of VAR-identified uncertainty shock with structural uncertainty shock:

[9] The simulations of the BB model are carried out with the replication files available from *Econometrica*'s website. Stephen Terry generously provided simulations of the RUBC model.

BB	RUBC	RSBC
0.94	0.94	0.91

Our identification scheme robustly identifies the true structural innovation to uncertainty across three very different structural models. Intuitively, that happens because in all three cases the identifying assumptions hold: the uncertainty shocks in the models all increase the future volatility of stock returns, and they have approximately zero effect on realized volatility on impact. To see that, we note that the correlations of the uncertainty shocks with realized volatility in the BB, RUBC, and RSBC models are all close to zero:

Contemporaneous correlation of structural uncertainty shocks with realized volatility:

BB	RUBC	RSBC
-0.01	-0.10	0.00

A reason that it is important to control for RV first in the VAR is that RV itself may raise future uncertainty. That effect is a standard result in the finance literature on volatility, often referred to as a GARCH (generalized autoregressive conditional heteroskedasticity) effect (Engle (1982), Bollerslev (1986)). We can also use the simulations of the models to explore what happens if the assumed timing of the shocks is reversed, failing to control for realized volatility in identifying the uncertainty shock. That is, what if we assume that uncertainty shocks move first and realized volatility second? In that case, the identifying assumption is that the change in uncertainty is affected by no other shocks in the economy. In the three simulated models, we then find that the VAR-identified uncertainty shock is less strongly correlated with the true uncertainty shock. Specifically, in the BB, RUBC, and RSBC models, the correlations between the VAR-identified and the true uncertainty shock become 0.94, 0.66, and 0.83, respectively. In the BB model, then, we can see that the timing is not relevant – that is because in the VAR in that model innovations to realized volatility and uncertainty are uncorrelated (which we will see below is strongly at odds with the data). In the other two models, though, realized volatility and uncertainty are more correlated, and the ordering of the variables does in fact matter. Given the robustness of GARCH effects, not surprisingly, we will find empirically that ordering is relevant.

To summarize, then, our identification scheme does extremely well in capturing the true uncertainty shock across the three models; conversely, none of the shocks in the reverse ordering correctly captures the uncertainty shock in general. That said, we will report results using both orderings in our empirical analysis below, and show that the results we obtain are both consistent with our interpretation of the facts.

4 Data

4.1 Macroeconomic data

As in other work in this literature (Bloom (2009), Leduc and Liu (2016)), we use monthly data to maximize statistical power, especially since fluctuations in both expected and realized volatility are sometimes short-lived (e.g. Bloom (2009)). We measure real activity using the Federal Reserve's measure of industrial production (IP) for the manufacturing sector. Employment and hours worked are measured as those of the total private non-farm economy.

4.2 Information about uncertainty

Obviously in order to identify a news shock, the vector of state variables in the VAR, Y_t, must contain information that can reveal expectations of future volatility beyond what is contained in current RV_t (u_t must have a component independent of the innovation to RV_t). We therefore include information from financial markets. First, we use $V_{1,t}$, the option-implied volatility of stock returns over the next month (we define $V_{1,t}$ very similarly to the VIX, as explained in the next section). Since $V_{1,t}$ may not include all the available financial information about uncertainty, we also include the six-month implied volatility, $V_{6,t}$, in some specifications.

Importantly, there is no assumption here that risk premia are zero or constant or that the option-implied volatility is measured without error. The only assumption that we need to calculate the impulse responses defined above is that some elements of Y_t contain information about future values of RV beyond the innovation to RV_t itself. We include option-implied volatilities because we would expect them to contain such information, but they are obviously also contaminated by risk premia and potential measurement error (e.g. due to stale prices or bid/ask spreads). Below we examine a number of other variables that past work has found can also help forecast volatility, but we find that their predictive power is subsumed by that in lagged RV and V_1.

Duffee (2011) shows that in standard linear term structure models, except for in knife-edge cases (even allowing for time-varying risk premia), investor expectations of the future can be extracted from observed asset prices. In our setting, that result corresponds to the view that the current state of the term structure of option-implied volatilities and RV_t should, together, encode all available information about future values of RV and option-implied volatility. We show that idea is in fact consistent with our data (even though Duffee (2011) argues it appears to be violated in the bond market), which will allow us to impose useful coefficient restrictions on the VAR and further reduce overfitting.

4.3 Financial data

We obtain data on daily stock returns of the S&P 500 index from the CRSP database and use it to construct RV_t at the monthly frequency. Option-implied volatilities, $V_{n,t}$, are constructed using prices of S&P 500 options obtained from the Chicago Mercantile Exchange (CME), with traded maturities from one to at least six months since 1983 (we thus have a substantially longer sample of implied volatility than has been used in past work). Given that shocks to stock market volatility are typically short-lived, with half lives often estimated to be on the order of six to nine months (see Bloom (2009) and Drechsler and Yaron (2011)), one- to six-month options will contain information about the dominant shocks to stock market uncertainty. Technically, $V_{n,t}$ is defined as the option-implied variance of the stock index on date $t+n$ under the pricing measure Q,[10]

$$V_{n,t} \equiv Var_t^Q \left[s_{t+n} \right] \tag{10}$$

The appendix provides the details of the calculation.

We use the measure of $Var_t^Q \left[s_{t+n} \right]$ in (10) because it maps directly to our object of interest, the conditional variance of log stock prices. In practice, it is extremely close to the VIX calculated by the CBOE and other related model-free implied volatility measures.

In the remainder of the paper we focus on the logs of realized and option-implied volatility ($rv_t \equiv \log RV_t$, $v_{n,t} \equiv V_{n,t}$) due to their high skewness, but we find similar results in levels.

4.4 The time series of uncertainty and realized volatility

Figure 1 plots the history of realized volatility along with 1-month option-implied volatility in annualized standard deviation terms. Both realized and option-implied volatility vary considerably over the sample. The two most notable jumps in volatility are the financial crisis in 2008 and the 1987 market crash, which both involved realized volatility above 75 annualized percentage points and rises of $V_{1,t}$ to 65 percent. At lower frequencies, the periods 1997–2003 and 2008–2012 are associated with persistently high uncertainty, while it is lower in other periods, especially the early 1980's, early 1990's, and mid-2000's. There are also distinct spikes in uncertainty in the summers of 2010 and 2011, likely due to concerns about the stability of the Euro zone and the willingness of the United States government to continue to pay its debts.

5 Second-moment forecasting regressions

Since identification of the second-moment news shock depends on using the variables in the VAR to forecast future realized volatility (i.e. through equation (2)), a natural first question is which of those variables, if any, has forecasting power for rv. Table 2 reports results of regressions of $\sum_{j=1}^{6} rv_{t+j}$ on various predictors. The first column reports results from a regression on rv_t and $v_{1,t}$. $v_{1,t}$ has a substantially larger t-statistic, indicating that it has greater explanatory power. The marginal R^2 of $v_{1,t}$ is 0.0554, while that of rv_t is smaller by a factor of eight at 0.0069. rv_t is in fact only marginally significant at the ten-percent level. In other words, option-implied volatility is a substantially stronger predictor of future realized volatility than the current level of realized

[10]The pricing measure, Q, is equal to the true (or physical) pricing measure multiplied by $M_{t+1}/E_t \left[M_{t+1} \right]$, where M_{t+1} is the pricing kernel.

volatility is. Given that asset prices are expected to aggregate information efficiently, it is not surprising that $v_{1,t}$ nearly drives rv_t out in a forecasting regression.

The second column of table 2 shows that we obtain a similar result when we include a lagged value of rv. In fact, if we include 6 lags of rv, their combined marginal R^2 is still only half the marginal R^2 of $v_{1,t}$. So in simple forecasting regressions, option prices yield substantially better predictions of future volatility than the past history of volatility does.

The third column of table 2 adds the six-month implied volatility, $v_{6,t}$, and finds that it adds no incremental information. We obtain similar results using a principal component of the term structure of implied volatilities. The fourth and fifth columns of table 2 add macroeconomic and financial variables to the regressions. None of them are individually statistically significant, nor are they jointly significant. The fifth column includes principal components from the large set of financial and macroeconomic time series collected by Ludvigson and Ng (2007), as well as the market return and the default spread (difference between the yields of Baa and Aaa corporate bonds). None of them has statistically significant forecasting power after controlling for rv_t and $v_{1,t}$, so we exclude them from the remainder of the analysis.

The R^2s are similar across all the specifications, and always 0.46 or less. The majority of the variation in six-month realized stock market volatility is thus unpredictable, even given information available at the beginning of the period.

It is perhaps notable that we do not find any variables beyond lagged rv and v_1 to be significant in predicting future volatility. Table A.1 in the appendix shows that when rv and v_1 are excluded from the regression, a number of the macroeconomic and financial variables become significant predictors of future volatility. In other words, the macro and financial time series on their own can help predict future volatility, but their forecasting power is subsumed by current realized and option-implied volatility.

6 VAR results

We now report the results from our main VAR. For all the VARs that we run, we include four lags, as suggested by the Akaike information criterion for our main specification. In the benchmark specification, the vector of variables included in the VAR is $[rv_t, v_{1,t}, FFR_t, ip_t, emp_t]$, where the latter three variables are the Fed Funds rate, log industrial production, and log employment, respectively.

The benchmark specification imposes the restriction that the coefficients on the lags of FFR, ip, and emp in the equations for rv and v_1 are equal to zero, consistent with the predictive regression results above. A χ^2 test of the validity of those restrictions yields a p-value of 0.61, implying that they are consistent with the data. Table A.2 in the appendix shows, in regressions analogous to those in table 2, that v_1 is predicted only by its own lags and those of rv (again consistent with the theoretical predictions of Duffee (2011)). In addition to their economic motivation, the restrictions help keep the news shock from being overfit, which we show can be a problem in section 6.3.2. These restrictions together imply that in the benchmark specification the uncertainty shock is equivalent to the reduced-form shock to v_1, orthogonalized with respect to rv. We relax the restrictions below and show that our results remain similar, though statistically weaker.

6.1 Coefficient estimates

Before reporting impulse responses it is useful to examine the coefficients in the VAR, similar to the regression results in table 1.[11] In our benchmark specification, the sums of the coefficients on lags of rv_t and $v_{1,t}$ in the equations for employment and industrial production are

	Sum of coefficients			
Dep. var.:	rv	v_1	Diff.	p-value
Employment	-0.0017	0.0012	0.0028	0.063
Indus. prod.	-0.0034	0.0024	0.0057	0.285

The coefficients on rv are negative for both variables, while the coefficients on v_1 are again positive. That basic result mirrors those in table 1 and appears consistently across the specifications that we estimate, giving a simple indication of the different effects of realized and expected volatility on real outcomes.

6.2 Benchmark specification

We now examine impulse response functions (IRFs), which describe the average responses of the variables in the economy to the two innovations. As discussed above, the IRFs are scaled so that the two shocks – current rv and the identified uncertainty shock – have the same total effect on volatility expectations (uncertainty) 2–24 months in the future (i.e. not counting the impact period).

Figure 2 has three columns for the responses of rv, employment, and industrial production to the shocks. Each panel shows the point estimate for the IRFs along with 1-standard deviation (68-percent) and 90-percent bootstrapped confidence intervals.

The first row shows the response of the economy to the rv shock. The shock to realized volatility is very short-lived: the IRF falls by half within two months, and by three-fourths within five months, showing that realized volatility has a highly transitory component. Those transitory increases in realized volatility are associated with statistically and economically significant declines in both employment and industrial production. So, consistent with past work, we find a significant negative relationship between *volatility* and real activity. However, this result does not allow us to conclude that an uncertainty shock is contractionary. The reason is that the realized volatility shock is a combination of an uncertainty shock (we can see from the first panel that the shock does

[11] These coefficients differ from the ones in table 1 because that regression includes the contemporaneous rv and v_1, whereas in this one only lags are included, according to the VAR specification. Also, the VAR includes lags of the macroeconomic variables.

predict future rv after impact, so it contains news about future volatility) with a shock to *current* realized volatility; by observing how the economy reacts to this combination of shocks we cannot draw conclusions about how it responds to the uncertainty shock only.

The second row of panels in figure 2 plots IRFs for the identified uncertainty shock, which has no contemporaneous effect on rv, but captures uncertainty about the future. First, as we would expect from equation (4), the news shock forecasts high realized volatility in the future at a high level of statistical significance (p-values testing whether the IRF is positive range between 0.001 and 0.05). That result alone is important: it says that the identified news shock contains statistically significant news about uncertainty.

Surprisingly, though, the second-moment news shocks are associated with no significant change in either employment or industrial production; the IRFs both stay very close to zero at all horizons. And the confidence bands are reasonably narrow: at almost all horizons, the point estimate for the responses of employment and industrial production to the rv shock are outside the 90-percent confidence bands for the uncertainty shock.

To further examine the magnitudes, the bottom row of panels in figure 2 reports the difference in the IRFs for the uncertainty and realized volatility shocks along with confidence bands. The two shocks have the same cumulative impacts on the future path of realized volatility (by construction, due to the scaling of the IRFs). But they obviously have different effects on rv on impact due to the identifying assumptions.

The two other panels in the bottom row of figure 2 plot the difference between the IRFs for industrial production and employment. We see that the difference is statistically significant for employment, weakly so for industrial production. So innovations in rv are followed by statistically significant declines in real activity, while uncertainty shocks are not, and that difference itself is statistically significant in one case.

An alternative way to interpret the bottom row of IRFs is that it represents the response of the economy to a pure shock to realized volatility that has no net effect on forward-looking uncertainty. By construction, the shock in the third row has a positive effect on rv on impact, but the IRF for rv over the following 24 periods sums to zero. The figure shows that such a shock has negative effects on the economy.

Overall, then, figure 2 shows that under our baseline identification scheme, shocks to rv are associated with statistically significant subsequent declines in output, while uncertainty shocks are not, and the difference between those two results is itself statistically significant. That result is notable given that past work (e.g. Bloom (2009), Basu and Bundick (2017), Leduc and Liu (2016)) has found that increases in option-implied volatility are followed by declines in output (a result we also obtain in our data; see appendix figure A.5). The results here show that it is actually *realizations* of volatility that seem to drive such effects, as opposed to shocks to uncertainty about the future. However, since realized volatility and option-implied volatility are positively correlated, using either of the two variables alone in a VAR as in past studies will lead the researcher to conclude that each shock is contractionary. To uncover the differential macroeconomic effects of realization and uncertainty shocks it is critical to include both in the VAR.

6.2.1 Volatility and uncertainty

To further understand the importance of the uncertainty and rv shocks, figure 3 reports forecast error variance decompositions (FEVDs). As in figure 2, we report the effect of the rv shock, the uncertainty shock, and their difference.

The first column reports variance decompositions for $E_t\left[\sum_{j=1}^{24} rv_{t+j}\right]$, which is our measure of uncertainty. The plots show that at the point estimates the rv shock accounts for 65 percent of the variance of uncertainty, while the uncertainty shock accounts for the remaining 35 percent. The confidence bands are wide, though: at most horizons, we cannot reject the hypothesis that the rv and news shocks account for the same fraction of the variance of uncertainty at even the 32-percent significance level. Since in the benchmark model the only variable that is assumed to have predictive power for uncertainty beyond rv_t itself is $v_{1,t}$, this is our most conservative specification and provides a lower bound on the fraction of variation in uncertainty coming from news shocks. Even in this case, though, the news shock accounts for a substantial fraction of the total variation in uncertainty.

To further examine the relative importance of the two shocks for uncertainty, figure 4 plots fitted uncertainty from the VAR along with the parts driven by the rv and identified uncertainty shocks. Total uncertainty is calculated from the VAR as $E_t\left[\sum_{j=1}^{24} rv_{t+j}\right]$. Note that in the VAR, there exists a vector B such that

$$E_t\left[\sum_{j=1}^{24} rv_{t+j}\right] = B\left[\tilde{Y}_t', \tilde{Y}_{t-1}', ..., \tilde{Y}_{t-4}'\right]' \tag{11}$$

$$\text{where } \tilde{Y}_t \equiv \left[Y_t', RV_t\right]' \tag{12}$$

To find the part of uncertainty driven by the rv shock, then, we construct a vector time series $\tilde{Y}^{rv}$ using the VAR structure setting all the estimated shocks to zero except for the rv shock.[12] Similarly, $\tilde{Y}^{news}$ is constructed by setting all estimated shocks except the news shock to zero. The parts of uncertainty coming from rv and news are then $B\tilde{Y}^{rv}$ and $B\tilde{Y}^{news}$, respectively. Ignoring constants, total uncertainty is equal to the sum of those two parts (which follows from the linearity of the model). The top panel of Figure 4 plots total uncertainty and the parts from rv and news (all demeaned).

The standard deviation of the part of uncertainty coming from news is only 22 percent smaller than that coming from rv, which is visible from the similar overall variation in the two series. The quarters with the largest news shocks are associated with events that are clearly associated with uncertainty: the 1987 and 2008 market crashes, and the third quarter of 2011 (debt ceiling and Euro crisis). At lower frequencies, the news-driven component of uncertainty is high around the

[12] That is, if $\tilde{Y}_t = C + F(L)\tilde{Y}_{t-1} + A\varepsilon_t$, we define ε^{rv} to be equal to the fitted values of ε but with all elements except for the first set to zero. $\tilde{Y}^{rv}$ is then constructed as $\tilde{Y}^{rv} = C + F(L)\tilde{Y}_{t-1}^{rv} + A\varepsilon_t^{rv}$.

1990 recession (i.e. around the First Gulf War), in the late 1990's (Asian financial crisis, Russian default, LTCM), and following the financial crisis (debt ceiling debates, Euro crisis).

To further evaluate the behavior of the time series of uncertainty implied by the model, the table below reports the correlation between the three series in figure 4 and three other commonly used measures of uncertainty: the Economic Policy Uncertainty index of Baker, Bloom, and Davis (EPU; 2015), a measure of uncertainty from the Michigan Consumer Survey (used by Leduc and Liu (2016)), and forecast uncertainty measure from Jurado, Ludvigson, and Ng (JLN; 2015).

Correlations with uncertainty indexes:

	EPU	Michigan	JLN
Total uncertainty	0.41	0.26	0.51
rv-driven uncertainty	0.40	0.22	0.76
News-driven uncertainty	0.09	0.12	-0.25

Our overall uncertainty measure is positively correlated with all three of the other measures of uncertainty, and that correlation is stronger for the rv-driven part. The pure uncertainty shocks that we identify here, that are separate from realized volatility, are essentially uncorrelated with the other uncertainty measures and therefore seem to capture feature of the data that is independent of them.

That said, and crucially for our analysis, the news shock is also not just noise. Again, it forecasts realized volatility significantly into the future, accounting for a substantial fraction of the variation in total uncertainty. Moreover, the identified news shock is strongly correlated with returns on the S&P 500. Specifically, the correlations of the identified rv and uncertainty shocks with S&P 500 returns are:

Correlations with S&P 500 returns:

rv shock:	-0.412
Uncertainty shock:	-0.405

That is, the correlations with stock returns are almost exactly the same. Both rv and the uncertainty shock (which are uncorrelated with each other by construction) have substantial negative correlations with stock returns. So not only is the uncertainty shock significantly associated with future volatility, but it is also associated with substantial declines in stock returns.[13] These facts make it all the more surprising that the uncertainty shock is not associated with any significant change in real activity.

6.2.2 Forecast error variance decompositions for employment and IP

In addition to the FEVDs for uncertainty, figure 3 also reports FEVDs for the two real outcomes, employment and industrial production. The realized volatility shock explains 25 percent of the

[13]Note that this result is entirely consistent with our identification scheme. The assumption is not that uncertainty shocks are uncorrelated with returns, but with realized volatility, which is a quadratic function of returns.

variance of employment and 10 percent of the variance of industrial production at the two-year horizon, while the point estimates for the fraction of the variance accounted for by second-moment news are two percent or less, and the upper ends of the 90-percent confidence intervals are near 5 percent for the first year. The upper end of the 90-percent confidence interval for the rv shock, though, reaches as high as 45 percent for employment and 30 percent for industrial production 24 months ahead, indicating that realized volatility can potentially be an important driver of the real economy (though this is *not* a causal statement since realized volatility is an equilibrium object).

Note also that the lack of importance of the uncertainty shock for real outcomes is not simply due to its size. Again, the pure uncertainty shock in the second row accounts for 35 percent of the total variance of uncertainty. So if we scale up the variance decompositions for employment and industrial production by multiplying them by 3, the point estimates still say that all uncertainty variation accounts for less than 5 percent of the total variance of employment and industrial production.

Looking at the third row, we see that the behavior of the rv and news shocks is again significantly different. For employment, the variance accounted for by the rv shock is larger at the 90-percent level, while for IP, they differ at only a 1-standard-deviation level.

In the end, then, figure 3 shows that pure uncertainty shocks account for a substantial fraction of the total variation in uncertainty about future stock returns, but they account for only a trivial fraction of the variation in real activity.

6.2.3 Robustness

Figures 5 and 6 report impulse responses for employment and IP to the two identified shocks and their difference across a number of perturbations of the benchmark specification.

First, we consider alternative orderings of the variables in the VAR. The effects of the ordering depend ultimately on the correlation matrix of the innovations, which we report below:

	rv	v_1	Fed Funds	Empl.	IP
rv	1				
v_1	0.73	1			
Fed Funds	0.01	-0.06	1		
Empl.	0.01	0.05	0.05	1	
IP	-0.04	0.01	0.04	0.54	1

The shocks to rv and v_1 are correlated with each other, but almost completely uncorrelated with those in the other variables, implying that if the news shock were orthogonalized not just to the shock to rv_t but to all the macro variables also, its IRF would remain unchanged, which the top panels of figures confirm by ordering rv and v_1 last.

The second row of panels in figures 5 and 6 reports results when we substitute v_6 for v_1. The six-month option-implied volatility seems like potentially a more natural variable to include since it might represent a more realistic economic decision horizon. As the figure shows, in that case we find results even less favorable to uncertainty shocks, with these shocks now having slightly expansionary effects on employment and industrial production.

The goal of the main analysis is to identify a pure shock to uncertainty that has no contemporaneous impact on realized volatility. A natural question is what happens if we reverse the ordering of the identification so that the first shock is the *entire* shock to uncertainty, while the second captures the residual variation in rv. In this case, then, the first shock is a combined shock to uncertainty and rv, while the second shock is a pure shock to realized volatility that has no net effect on uncertainty on impact. Figure A.6 reports results from such a specification. It shows that both these shocks have essentially the same effect on employment and industrial production, exactly as should be expected from our main analysis. Recall that in the main analysis, the identified rv and uncertainty shocks have similar effects on future volatility, but they are identified so that only the rv shock affects rv on impact. In figure A.6, on the other hand, the shocks have very *similar* effects on rv on impact, and they are now distinguished by having *different* effects on future volatility.[14]

Overall, then, when two shocks have the same initial effect on rv, they have the same effects on output (figure A.6) whereas when they have different effects on rv, they have different effects on output (figure 2). In other words, it is the impact of a shock on contemporaneous realized volatility, not its effect on uncertainty, that determines how it affects output.

Figures A.9 and A.10 report results from four additional robustness tests:

1. Replacing our volatility measure with the VIX and VXO
2. Limiting the sample to 1988–2006 to remove the jumps in volatility in 1987 and 2008.
3. Controlling for the S&P 500 in the VAR before the identified shocks.
4. Using RV and V_1 (i.e. in levels rather than logs).

The results of those robustness tests are qualitatively and quantitatively consistent with our baseline results.

Figures A.7 and A.8 report results from a quarterly VAR similar to what is estimated in Basu and Bundick (2016) with the set of variables now rv, v_1, GDP, aggregate consumption, aggregate investment (all in real terms), the GDP deflator, and the Fed funds rate. The large number of variables and smaller number of time series observations gives this VAR specification much lower power overall than our benchmark monthly specification. As in our benchmark results, output, consumption, and investment all fall following an increase in rv. However, in this case the confidence bands around the uncertainty shock are wide enough to render the estimates essentially uninformative. When we use v_1 to help measure uncertainty, the shock appears slightly contractionary, while using v_6 makes it appear actually expansionary. While the quarterly data does not yield sufficient power to draw firm conclusions, it in no way conflicts with our main results, and it reinforces the robustness of the findings for the contractionary effects of realized volatility shocks.

Finally, we also estimate a factor-augmented VAR using the first three principal components

[14] For robustness, we have also repeated the exercise of reversing the ordering of realized volatility and uncertainty using the VIX instead of our implied volatility measure. The results are in figure A.11 and are qualitatively and quantitatively consistent with the results of figure A.6.

from the set of macroeconomic time series studied by Ludvigson and Ng (2007). The advantage of the FAVAR specification is that incorporates information from an extremely broad range of variables, instead of just using employment and industrial production (we use the setup of Bernanke, Boivin, and Eliasz (2005)). Figure A.12 reports results from that estimation, which are again highly similar to the benchmark.

6.3 Allowing more variables to predict second moments

The benchmark VAR has two key restrictions that this subsection relaxes. First, it assumes that the vector $[rv_t, v_{1,t}]'$ is driven only by its own lags (that is, the coefficients in those rows of the VAR on the other variables are set to zero). Second, it uses information only from a single point on the term structure of implied volatilities. Those restrictions are both consistent with the data in terms of the regressions in table 2 and a χ^2 test in the VAR, but it is reasonable to ask what happens when they are relaxed. This section first uses lasso as an alternative method to reduce overfitting, and second allows v_6 to enter the VAR and removes all the coefficient restrictions. In that case, the uncertainty series appears to be substantially overfit, emphasizing why it is desirable to discipline how the news shock is constructed.

6.3.1 Lasso

Lasso (Tibshirani (1996)) is a regularization method for regressions. Instead of choosing the set of coefficients in the VAR just to minimize the residual variance, the objective function in lasso also includes a penalty on the sum of the absolute values of the coefficients. Because the penalization function is not differentiable at zero (with a positive slope on both sides of zero), it implies that coefficients that are sufficiently small are optimally set to zero. The advantage of using lasso for our purposes is that it is economically agnostic and driven by statistical considerations. It thus yields restrictions on the VAR to help reduce overfitting similarly to our benchmark specification, but without imposing the set of restrictions based purely on theory.

The appendix describes the details of our implementation of lasso. We choose the magnitude of the penalty on the coefficients based on a cross-validation criterion. In our implementation, lasso restricts many of the coefficients on the macro variables in the equations for rv and v_1 to zero, but not all of them. It also restricts the coefficients on some of the lags of rv and v_1 to be zero (they were not restricted in the benchmark). So it reduces the restrictions on the macro variables but increases them on the lags of the volatility measures.

The third row in figures 5 and 6 reports results using lasso for the estimation instead of the benchmark restrictions. The results remain highly similar to the benchmark case, showing that the restrictions applied in the benchmark model do not drive the results alone.

6.3.2 Completely unrestricted model

Finally, the bottom rows in figures 5 and 6 report results using the completely unrestricted identification scheme from BBL for the news shock and include $v_{6,t}$ as an additional predictor of volatility. These are thus our most general and least constrained results. Shocks to realized volatility continue to be contractionary, and we again find no evidence that the news shock has negative effects on output, but the confidence bands for the news shock in this case become so wide as to be uninformative. The difference between the IRFs for rv and news is now statistically insignificant. We obtain similar results when we include other predictors of future volatility instead of $v_{6,t}$ such as the default spread.

The unrestricted specification has low power for identifying the effect of news shocks because the news shock itself is difficult to identify. There are, in this case, 24 potential predictors, and noise in the estimated coefficients on them is inherited by the identified news shock. That is why the specifications with restrictions and using lasso, which both have fewer coefficients to estimate, are much more stable.

Beyond power, though, the unrestricted model is potentially problematic since the news shock may be overfit. That can be seen partly from the fact that the news shock in this case actually accounts for a substantially *larger* amount of the variance of uncertainty than the rv shock does – up to 60 percent of the total variance at short horizons.

As further evidence of overfitting, the bottom panel of figure 4 plots total uncertainty, $E_t\left[\sum_{j=1}^{24} rv_{t+j}\right]$, from the benchmark and completely unrestricted specifications. We see that in the unrestricted specification, uncertainty varies much more – its standard deviation is higher by a factor of 1.6. Moreover, it rises well in advance of the 2008 financial crisis, even though realized and option-implied volatility were low. That is, the unrestricted model seems to have seen the crisis coming before investors, consistent with overfitting. Uncertainty in the restricted model leads the benchmark specifications in a number of other episodes. This lead-lag relationship can also be confirmed by examining cross-correlations. Finally, whereas uncertainty in the restricted model is positively correlated with alternative measures of uncertainty, the Baker–Bloom–Davis index and the Michigan index, uncertainty in the unrestricted model is actually *negatively* correlated with those series (with coefficients of -0.25 and -0.30), again emphasizing the implausibility of this specification.

In the end, then, when we use the most general specification and add no information to the model to help control the predictive coefficients, we continue to find no evidence that uncertainty shocks are contractionary, but we no longer have power to statistically distinguish their effects from those of rv shocks. The most conservative interpretation of our empirical analysis is therefore simply that *realized* volatility is followed by contractions and that it is critical to separate realized volatility from uncertainty when trying to estimate the effects of uncertainty shocks. In the specifications that we view as most reasonable, though, which add information to the VAR either through economic priors or through statistical regularization, we can go a step further and argue that there is in fact some affirmative evidence that uncertainty shocks have effects on the economy that are quantitatively close to zero, in terms of both impulse responses and forecast error variance decompositions.

7 Evidence from risk premia

We now show that investors have historically paid large premia for insurance against increases in realized volatility, but not for insurance against increases in market-implied uncertainty, which suggests that investors do not view periods in which uncertainty rises as having high marginal utility (i.e. as being bad times), consistent with our VAR results. This fact has been established in past work; here we review the evidence and extend the time series further back than in other analyses.[15]

A one-month variance swap is an asset whose final payoff is the sum of daily squared log returns of the underlying index (the S&P 500, in our case) over the next month. That asset gives the buyer protection against a surprise in equity return volatility (rv) over the next month. If investors are averse to periods of high realized volatility, then, we would expect to see negative average returns on one-month variance swaps, reflecting the cost of buying that insurance. A simple way to see that is to note that in general the Sharpe ratio of an asset, the ratio of its expected excess return to its standard deviation, is

$$\frac{E_t\left[R_{t+1}-R_{f,t+1}\right]}{SD_t\left[R_{t+1}\right]}=-corr_t\left(R_{t+1},MU_{t+1}\right)\times std\left(MU_{t+1}\right) \tag{13}$$

for any return R_{t+1}, where $R_{f,t+1}$ is the risk-free rate and MU_{t+1} denotes the marginal utility of consumption on date $t+1$. Assets that covary positively with marginal utility, and hence are hedges, earn negative average returns. So if realized volatility is high in high marginal utility states (in most models, bad times), then one-month variance swaps will earn high Sharpe ratios.

The first point on the left in the left-hand panel of Figure 7 (which is drawn from Dew-Becker et al. (2017)) plots average annualized Sharpe ratios on 1-month S&P 500 variance swaps between 1996 and 2014.[16] The average Sharpe ratio is -1.4, approximately three times larger (with the opposite sign) than the Sharpe ratio on the aggregate equity market in that period. In other words, investors have been willing to pay extraordinarily large premia for protection against periods of high realized volatility, suggesting that they view those times as particularly bad (or as having very high marginal utility).

Now consider a j-month variance forward, whose payoff, instead of being the sum of squared returns over the next month ($t+1$), is the sum of squared returns in month $t+j$ (so then the one-month variance swap above can also be called a 1-month variance forward). If an investor buys a

[15] See for example Egloff, Leippold, and Wu (2010); Ait-Sahalia et al. (2015); Dew-Becker et al. (2017). A large literature in finance studies the pricing of realized and expected future volatility. See, among many others, Adrian and Rosenberg (2008), Bollerslev et al. (2009), Heston (1993), Ang et al. (2006), Carr and Wu (2009), Bakshi and Kapadia (2003), Egloff, Leippold, and Wu (2010), and Ait-Sahalia, Karaman, and Mancini (2013) (see Dew-Becker et al. (2017) for a review).

[16] The data is described in Dew-Becker et al. (2017); it is obtained from a large asset manager and Markit, but may be closely approximated by portfolios of options, for which prices are widely available (e.g. from Optionmetrics).

j-month variance forward and holds it for a single month, selling it in month $t+1$, then the variance forward protects her over that period against *news* about volatility in month $t+j$. If between t and $t+1$ investors receive news that volatility will be higher in the future – i.e. if uncertainty rises – the holding period return on that j-period forward will increase. The left-hand panel of figure 7 also plots one-month holding period Sharpe ratios for variance forwards with maturities from 2 to 12 months. We see that for all maturities higher than 2 months, the Sharpe ratios are near zero, and in fact the sample point estimates are positive. The Sharpe ratios are also all statistically significantly closer to zero than the Sharpe ratio on the one-month variance swap.

The left panel of Figure 7 therefore shows that investors have paid large premia for protection against surprises in realized volatility, but news about future uncertainty has had a premium that is indistinguishable from zero, and may even be positive. Realized volatility thus appears to have a large positive correlation with marginal utility, while shocks to expected volatility have a correlation that is close to zero or negative.

Using the options data described above, it is possible to extend those results further, back to 1983. The right-hand panel of figure 7 reports the average shape of the term structure of variance forward prices constructed from data on S&P 500 options (we study the term structure with this data because it is estimated more accurately than returns). The variance forwards are constructed from synthetic variance swaps, a calculation almost identical to our calculation of $Var_t^Q[s_{t+n}]$. The term structure reported here is directly informative about risk premia. The average return on an n-month variance claim is:

$$\underbrace{E\left[\frac{F_{n-1,t}-F_{n,t-1}}{F_{n,t-1}}\right]}_{\text{Average return of the forward}} \approx \frac{\overbrace{E\left[F_{n-1}\right]-E\left[F_{n}\right]}^{\text{Average slope of the forward curve}}}{E\left[F_{n}\right]} \tag{14}$$

where $F_{n,t}$ is the price on date t of an n-maturity volatility forward. The slope of the average term structure thus indicates the average risk premium on news about volatility n months forward. If the term structure is upward sloping, then the prices of the variance claims fall on average as their maturities approach, indicating that they have negative average returns. If it slopes down, then average returns are positive.

The right-hand panel of Figure 7 plots the average term structure of variance forward prices for the period 1983–2013. The term structure is strongly upward sloping for the first two months, again indicating that investors have paid large premia for assets that are exposed to realized variance and expected variance one month in the future. But the curve quickly flattens, indicating that the risk premia for exposure to fluctuations in expected variance further in the future have been much smaller.

The asset return data says that investors appear to have been highly averse to news about high realized volatility, while shocks to expected volatility do not seem to have been related to marginal utility. Figure 7 therefore confirms and complements the results from our VAR, that show that shocks to rv are associated with recessions but uncertainty shocks are not.

8 Equilibrium model and further evidence

The paper thus far has provided empirical evidence on two basic points: first, surprises in realized volatility in the stock market are associated with future declines in real activity, while uncertainty shocks, identified as second-moment news, are not; second, investors have historically paid large premia to hedge shocks to realized volatility, but have paid premia that have averaged to nearly zero to hedge shocks to uncertainty.

In this section we present a simple stylized structural model that is consistent with those features of the data. The key ingredient is asymmetry in fundamental shocks. Intuitively, when fundamental shocks are skewed to the left, large shocks, which are associated with high realized volatility, tend to be negative.[17] That is simply the definition of left skewness: the squared innovation is negatively correlated with the level of the innovation. We first discuss evidence that there is left skewness in economic activity and then describe the model and show that estimated impulse responses to uncertainty and realized volatility shocks in the model match what we find in the data.

8.1 Skewness

A potential source of negative correlation between output and realized volatility is negatively skewed shocks. Specifically, if some shock ε is negatively skewed, then $E\left[\varepsilon^3\right] < 0 \Rightarrow cov\left(\varepsilon, \varepsilon^2\right) < 0$. There are large literatures studying skewness in both aggregate stock returns and economic growth. We therefore provide a brief overview of the literature and basic evidence.

Table 3 reports the skewness of monthly and quarterly changes in a range of measures of economic activity. Nearly all the variables are negatively skewed, at both the monthly and quarterly levels. One major exception is monthly growth in industrial production, but that result appears to be due to some large fluctuations in the 1950s. When the sample is cut off at 1960, the results for industrial production are consistent with those for other variables.

In addition to real variables, table 3 also reports realized and option-implied skewness for S&P 500 returns.[18] The implied and realized skewness of monthly stock returns is substantially negative and similar to the skewness of capacity utilization. The realized skewness of stock returns is less negative than option-implied skewness, which is consistent with investors demanding a risk premium on assets that have negative returns in periods when realized skewness is especially negative (i.e. that covary positively with skewness).

[17] Skewness in equilibrium quantities could arise because the fundamental shocks are skewed, or because symmetrical shocks are transmitted to the economy asymmetrically (perhaps because constraints, such as financial frictions, bind more tightly in bad times; see Kocherlakota (2000), or because firms respond to shocks in a concave manner, as in Ilut, Kehrig, and Schneider (2016)).

[18] We obtain option-implied skewness from the CBOE's time series of its SKEW index, which is defined as $SKEW = 100 - 10 \times Skew\left(R\right)$. We thus report $10 - SKEW/10$.

In addition to the basic evidence reported here, there is a large literature providing much more sophisticated analyses of asymmetries in the distributions of output and stock returns. Morley and Piger (2012) provide an extensive analysis of asymmetries in the business cycle and review the large literature. Estimating a wide range of models, they show that those that fit aggregate output best have explicit non-linearity and negative skewness. Even after averaging across models using a measure of posterior probability, they find that their measure of the business cycle is substantially skewed to the left, consistent with the results reported in table 3. More recently, Salgado, Guvenen, and Bloom (2016) provide evidence that left skewness is a robust feature of business cycles, at both the macro and micro levels and across many countries.

The finance literature has also long recognized that there is skewness in aggregate equity returns and in option-implied return distributions (Campbell and Hentschel (1992); Ait-Sahalia and Lo (1998); Bakshi, Kapadia, and Madan (2003)). The skewness that we measure here appears to be pervasive and has existed in returns reaching back even to the 19th century (Campbell and Hentschel (1992)).

Taken as a whole, then, across a range of data sources and estimation methods, there is a substantial body of evidence that fluctuations in the economy are negatively skewed.

8.2 An equilibrium model

We now present a simple extension to the RBC model consistent with our empirical findings that (1) shocks to realized volatility are associated with declines in real activity, while shocks to expected volatility are not; (2) Sharpe ratios on short-term claims to volatility are much more negative than those on longer-term claims; and (3) output growth and equity returns are negatively skewed. We deliberately keep the model simple in order to highlight the economic channels that are at work; it is not rich enough to provide a tight quantitative fit to the economy. But despite its simplicity, the model is qualitatively consistent with nearly all our results, and it provides further support for the VAR identification scheme.

8.3 Model structure

Firms produce output with technology, A_t, capital, K_t, and labor, N_t,

$$Y_t = A_t^{1-\alpha} K_t^{\alpha} N_t^{1-\alpha} \tag{15}$$

We set $\alpha = 0.33$, consistent with capital's share of income. Capital is produced subject to adjustment costs according to the production function

$$K_t = \left[(1-\delta) + \frac{I_{t-1}}{K_{t-1}^{agg}} - \frac{\zeta}{2} \left(\frac{I_{t-1}}{K_{t-1}^{agg}} - \overline{I/K} \right)^2 \right] K_{t-1} \exp\left(-J\nu_t\right) \tag{16}$$

where I_t is gross investment, K_t^{agg} is the aggregate capital stock (which is external to individual firm decisions), ζ is a parameter determining the magnitude of adjustment costs and $\overline{I/K}$ is the steady-state investment/capital ratio. The term $\exp(-J\nu_t)$ is associated with downward jumps in productivity that are described below – those jumps destroy both technology and capital, similar to Gourio (2012). We set $\delta = 0.08/12$ (corresponding to a monthly calibration) and $\zeta = 0.5$.[19] Given the structure for adjustment costs and production, the equilibrium price and return on a unit of installed capital are

$$P_{K,t} = \frac{1}{1-\zeta\left(\frac{I_t/K_{t-1}}{\overline{I/K}}-1\right)} \tag{17}$$

$$R_{K,t} = \frac{\alpha A_t^{1-\alpha}K_{t-1}^{\alpha-1}N_t^{1-\alpha}+(1-\delta)P_{K,t}}{P_{K,t-1}} \tag{18}$$

A representative agent maximizes Epstein–Zin (1991) preferences over consumption and leisure,

$$V_t = \arg\max_{C,N} \log\left(C_{t+j}-bC_{t+j-1}^{agg}\right)-\theta\frac{N_{t+j}^{1+\chi}}{1+\chi}+\beta\log E_t\exp\left(-\gamma V_{t+1}\right) \tag{19}$$

where C^{agg} is aggregate consumption, subject to the budget constraint

$$C_t + I_t \leq Y_t \tag{20}$$

Agents have log utility over consumption minus an external habit. We set the magnitude of the habit to $b = 0.8$ to help generate smoothness in consumption. β is set to $0.99^{1/12}$, χ to 1/3 for a Frisch elasticity of 3, and θ to generate steady-state employment of 1/3. The coefficient of relative risk aversion, γ, is set to 6 to generate a Sharpe ratio on a claim to capital of 0.35, similar to what is observed for US equities. Note, though, that the habit will induce variation in effective risk aversion over time.

The model is closed by the Euler equation and the optimization condition for labor,

$$1 = E_t\left[\beta\frac{\exp\left((1-\alpha)V_{t+1}\right)}{E_t\exp\left((1-\alpha)V_{t+1}\right)}\frac{C_t-bC_{t-1}^{agg}}{C_{t+1}-bC_t^{agg}}R_{K,t+1}\right] \tag{21}$$

$$\theta N_t^{\chi}\left(C_t-bC_{t-1}^{agg}\right) = (1-\alpha)A_t^{1-\alpha}K_{t-1}^{\alpha}N_t^{-\alpha} \tag{22}$$

We model realized volatility as in the empirical analysis as the square of the levered excess return on capital,

$$RV_t = \left(\lambda\left(R_{K,t+1}-R_{f,t}\right)+R_{f,t}\right)^2 \tag{23}$$

where $R_{f,t}$ is the risk-free rate and the parameter λ determines the leverage of equity. We set $\lambda = 9.2$

[19] See, e.g., Cummins, Hassett, and Hubbard (1994) for estimates of adjustment costs similar to this value. Jermann (1998) use a similar values. $\zeta = 0.5$ is on the lower end of estimates based on aggregate data and more consistent with micro evidence, but our results are not sensitive to the choice of this parameter.

to match the volatility of the aggregate stock market. It is then straightforward to construct prices on claims to future realized volatility.

The only exogenous variable in the model is technology, A_t, which follows the process

$$\Delta \log A_t = \sigma_{t-1}\bar{\sigma}_\varepsilon \varepsilon_t - J(\nu_t - \bar{p}) + \mu \tag{24}$$

$$\log \sigma_t = \phi_\sigma \log \sigma_{t-1} + \sigma_\sigma \eta_t + \kappa_{\sigma,\nu}\nu_t \tag{25}$$

$$\varepsilon_t, \eta_t \sim N(0,1) \tag{26}$$

$$\nu_t \sim Bernoulli(\bar{p}) \tag{27}$$

Technology follows a random walk in logs with drift, μ, set to 2 percent per year. ε_t is a normally distributed innovation that affects technology in each period, while ν_t is a shock that is equal to zero in most periods but equal to 1 with probability $\bar{p}$ – that is, it induces downward jumps in technology, with J determining the size of the jump and $\bar{p}$ the average frequency. σ_t determines the volatility of normally distributed shocks to technology. It is itself driven by two shocks: an independent shock η_t (volatility news) and also the jumps ν_t. Downward jumps in technology can be associated with higher volatility, generating ARCH-type effects (Engle (1982)). The volatility process thus has two features that will be important in matching the data: it has news shocks, and it is countercyclical for $\kappa_{\sigma,\nu} < 0$.

ϕ_σ and σ_σ are calibrated so that $\log \sigma_t$ has a standard deviation of 0.35 and a one-month autocorrelation of 0.91, consistent with the behavior of the VIX. $\kappa_{\sigma,\nu}$ is set to -0.7, which implies that a jump in technology increases σ_t by 2 standard deviations, generating countercyclical volatility. $\bar{\sigma}_\varepsilon$ is set so that normally distributed shocks on average generate a standard deviation of output growth close to the value of 1.92 we observe empirically. Jumps on average reduce technology by 8 percent (which is 3.2 times $\bar{\sigma}_\varepsilon$, the average standard deviation of the Gaussian TFP shocks) and are calibrated to occur once every 10 years on average. We thus think of them as representing small disasters or large recessions (consistent Backus, Chernov, and Martin (2011) and with the view of skewed recessions in Salgado, Guvenen, and Bloom (2016)), rather than depression-type disasters.[20] While the size of the jump seems large initially, recall that the model is calibrated to match the standard deviation of output growth.

We solve the model by projecting the decision rule for consumption on a set of Chebyshev polynomials up to the 6th order (a so-called global solution) to ensure accuracy not only for real dynamics but also for asset prices and realized volatility. Integrals are calculated using Gaussian quadrature with 20 points. Euler equation errors are less than $10^{-5.0}$ across the range of the state space that the simulation explores and have an average absolute value of $10^{-5.3}$. The use of a global solution method allows for high accuracy in the solution, but also makes it infeasible to search over many parameters or estimate the model, which is why we explore just a single calibration here.

[20] A realistic extension of the model would be to allow for jumps to be drawn from a distribution, rather than all having the same size. See, e.g., Barro and Jin (2011).

8.4 Simulation results

We examine three sets of implications of the model: VAR estimates, risk premia, and skewness. All results are population statistics calculated from a simulation lasting 10,000 years.

Table 4 reports basic moments of returns on capital and growth rates of output, consumption, and investment. The model generates negative skewness in all four variables in the table, consistent with the data, but the skewness is much larger than is observed empirically. Mean growth rates of real variables are similar to the data. The standard deviation of output is very close to the data, while consumption is more and investment less volatile; the gap between the two is smaller than observed empirically, however. Table 4 thus suggests that the model generates moments that are broadly consistent with the data, in particular generating comovement among aggregate variables (the three growth rate series in table 4 have correlations between 0.53 and 0.96) and volatilities that are empirically reasonable.

Figure 8 plots the Sharpe ratios of volatility claims in the model that correspond to the forward volatility claims examined in Figure 7. As in the left panel of Figure 7, the Sharpe ratio of the one-month asset, which is a claim to realized volatility, is far more negative than the Sharpe ratios for the claims with longer maturities. Intuitively, this is because shocks to volatility expectations, η_t, have relatively small effects on consumption and lifetime utility, hence earning a small risk premium. Shocks to realized volatility, on the other hand, tend to isolate the jumps, ν_t, (as we will show below), so they earn larger premia.

Finally, figure 9 plots IRFs from the estimation of our VAR in the simulation of the model. The gray shaded regions represent confidence bands from the VAR in the empirical data.[21] The red lines are the population IRFs from the VAR estimated in the simulated model. The IRFs here represent responses to unit standard deviation impulses to the identified shocks.[22]

The model matches well in terms of how an uncertainty shock predicts future stock market volatility and output. Most importantly, it generates a large and empirically reasonable decline in output following the realized volatility shock and an economically small response of output to the identified uncertainty shock. Both IRFs lie within the confidence bands at most horizons. The model performs more poorly in matching the behavior of employment. Employment responds little to either of the shocks, so it matches the empirical IRF for the news shock, but not the realized volatility shock.

The VAR results are notable because they replicate the behavior of output observed empirically even though there is no structural "realized volatility shock" in the model. Rather, the identified RV shock comes from the jump in TFP in the model ($J\nu_t$). To see that, we report the correlations

[21] These confidence bands are slightly different from what is reported in figure 2 because they use the levels of RV and option-implied volatility instead of their logs (since the model is in discrete time and realized volatility is the monthly squared return, it can approach zero, causing the log transform to break down).

[22] Note that this differs from the empirical analysis above, which scaled the realized volatility shock to have the same effect on uncertainty (expected future volatility) as the news shock. The reason for the difference here is that realized volatility in the model is a relatively weak predictor of future volatility, making the scaling sometimes explode to infinity.

between the VAR-identified shocks and the structural shocks in the model in the bottom section of table 4. The RV shock is correlated nearly exclusively with $J\nu_t$, the jump shock in the model. So the VAR successfully identifies the jumps as realized volatility shocks, which are then structurally, but obviously not causally, related to declines in output.

The identified uncertainty shock, as we would hope, is almost purely correlated with η_t, the volatility news shock. So, as discussed above, and similar to the BB and RUBC models, our main VAR specification does a good job in this setting – a non-linear production model – of actually identifying true structural shocks and also fitting the qualitative behavior of our empirical VAR analysis.

To further illustrate the mechanisms driving the model, the dotted blue lines in figure 9 plot the dynamic responses of the variables to the true structural shocks in the model. In the top row the dotted lines plot the responses to the jump shock, ν_t, while in the bottom row they plot the responses to the uncertainty shock, η_t (again in both cases scaled to be unit standard deviations). In four of the cases the response to the structural shock is essentially identical to the estimate from the VAR. The responses of output are slightly different (since the VAR is not a perfect representation of the structural model) but still very close to the VAR.

The fact that the response of employment to the jump shock is small is a consequence of the use of preferences consistent with balanced growth following King, Plosser, and Rebelo (1988). Since the jumps in technology also involve destruction of capital, they represent essentially a shift along the balanced growth path. In the absence of habit formation, they would have precisely zero effect on employment; the habit causes a slight positive response. Obtaining more negative responses of employment would require adding frictions or changing the preferences, which we leave to future work.

The results in this section show that two features of the model explain the behavior of the VAR in the simulations. First, the negative relationship between realized volatility and output comes from the fact that realized volatility is high in months with jumps, and all jumps are negative. Second, the lack of a relationship between output and uncertainty comes from the fact that the true uncertainty shock has essentially zero effect on output.

In the end, then, this section shows that a simple production model can match the basic features of the data that we have estimated in this paper: output responds negatively to shocks to realized volatility but not to shocks to uncertainty, there is a much larger risk premium for realized than expected volatility, and economic activity and stock returns are both skewed to the left.

9 Conclusion

The key distinction that this paper draws is between realized volatility and uncertainty. Volatility matters for output, but it is the realized part that is robustly followed by downturns. Changes in expected volatility – uncertainty shocks – appear to have no significant negative effects. Evidence from asset prices and risk premia is consistent with these findings, and we develop a simple model that can rationalize the data and also justifies our identification scheme.

The empirical results are inconsistent with theories in which pure shocks to aggregate uncertainty play an important role in driving real activity. Moreover, the identification scheme used in this paper is shown to successfully identify true structural shocks to uncertainty in leading recent models.

More constructively, this paper aims to lay out a specific view of the joint behavior of stock market volatility and the real economy. There appear to be negative shocks to the stock market that occur at business cycle frequencies, are associated with high realized volatility and declines in output, and are priced strongly by investors. The simple idea that fundamentals are skewed left can explain our VAR evidence, the pricing of volatility risk, and the negative unconditional correlation between economic activity and volatility.

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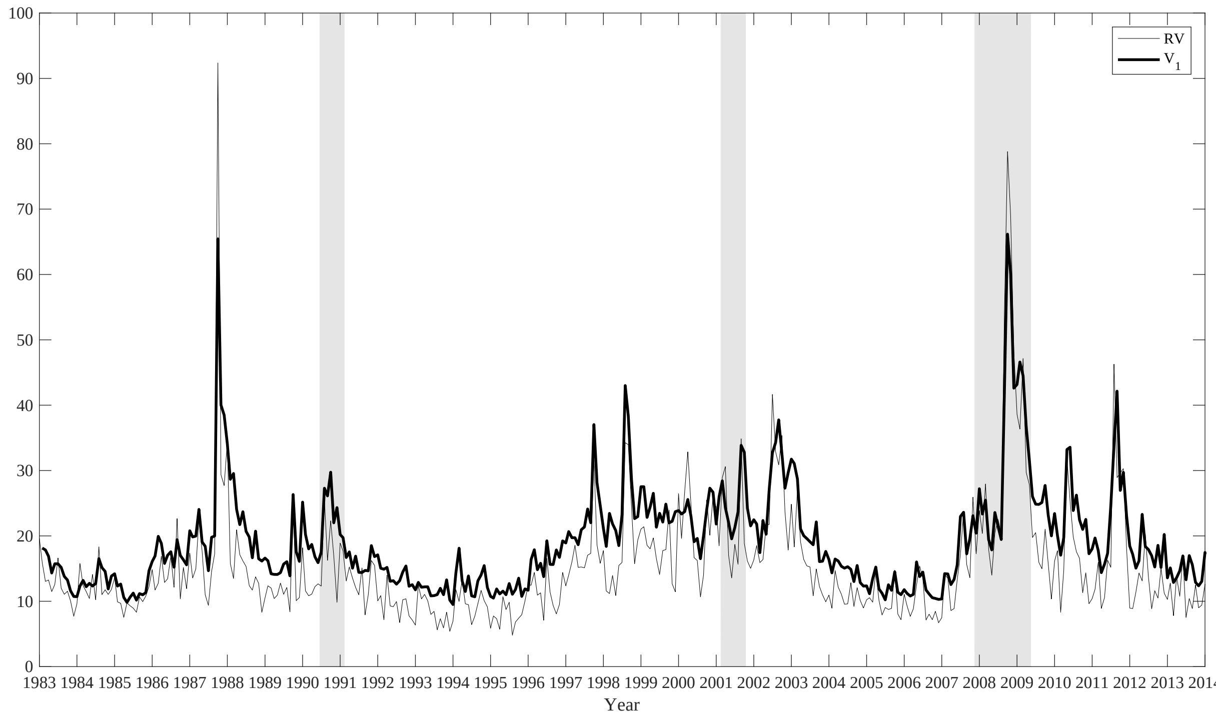
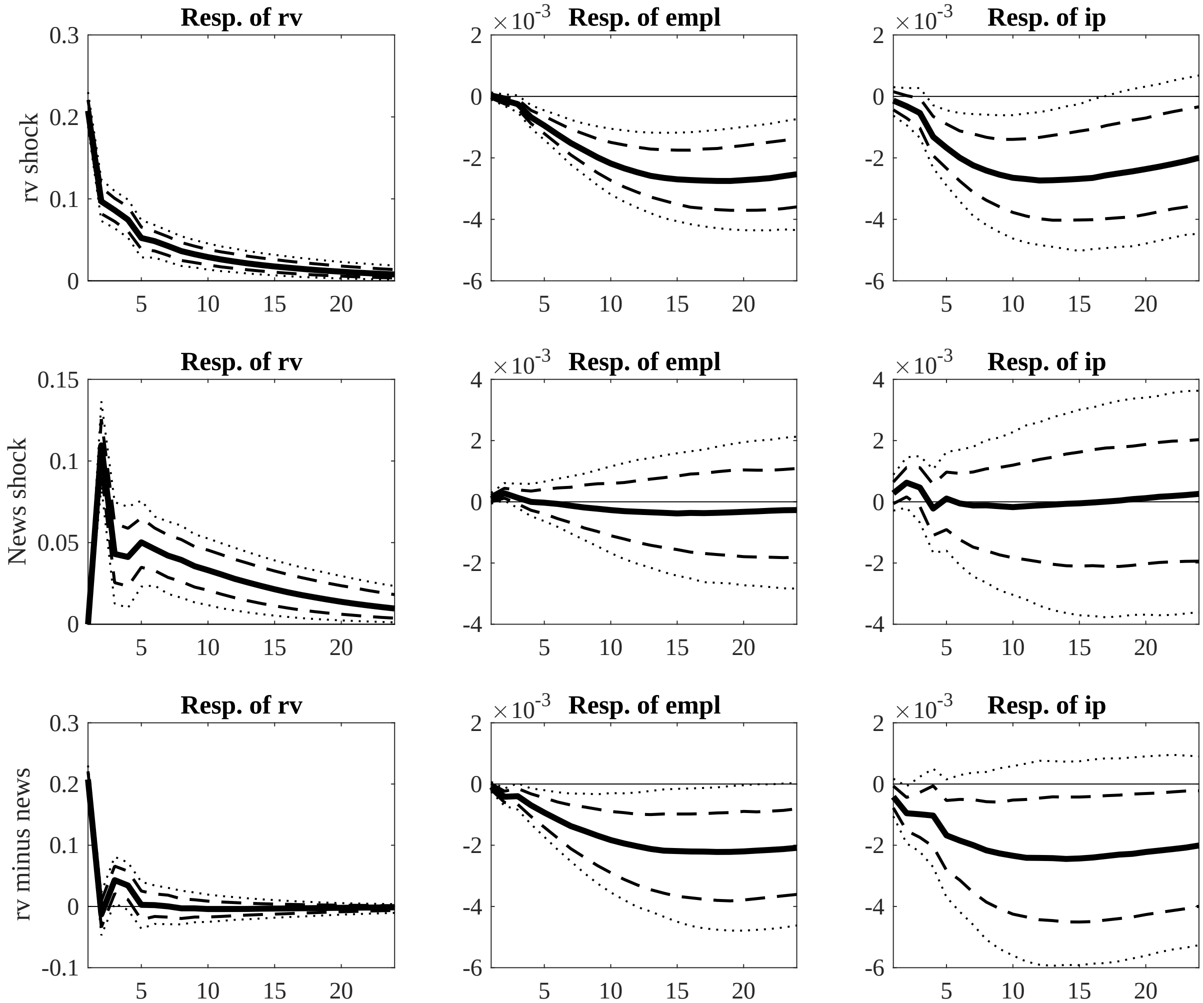
Figure 1: Time series of realized volatility and expectations



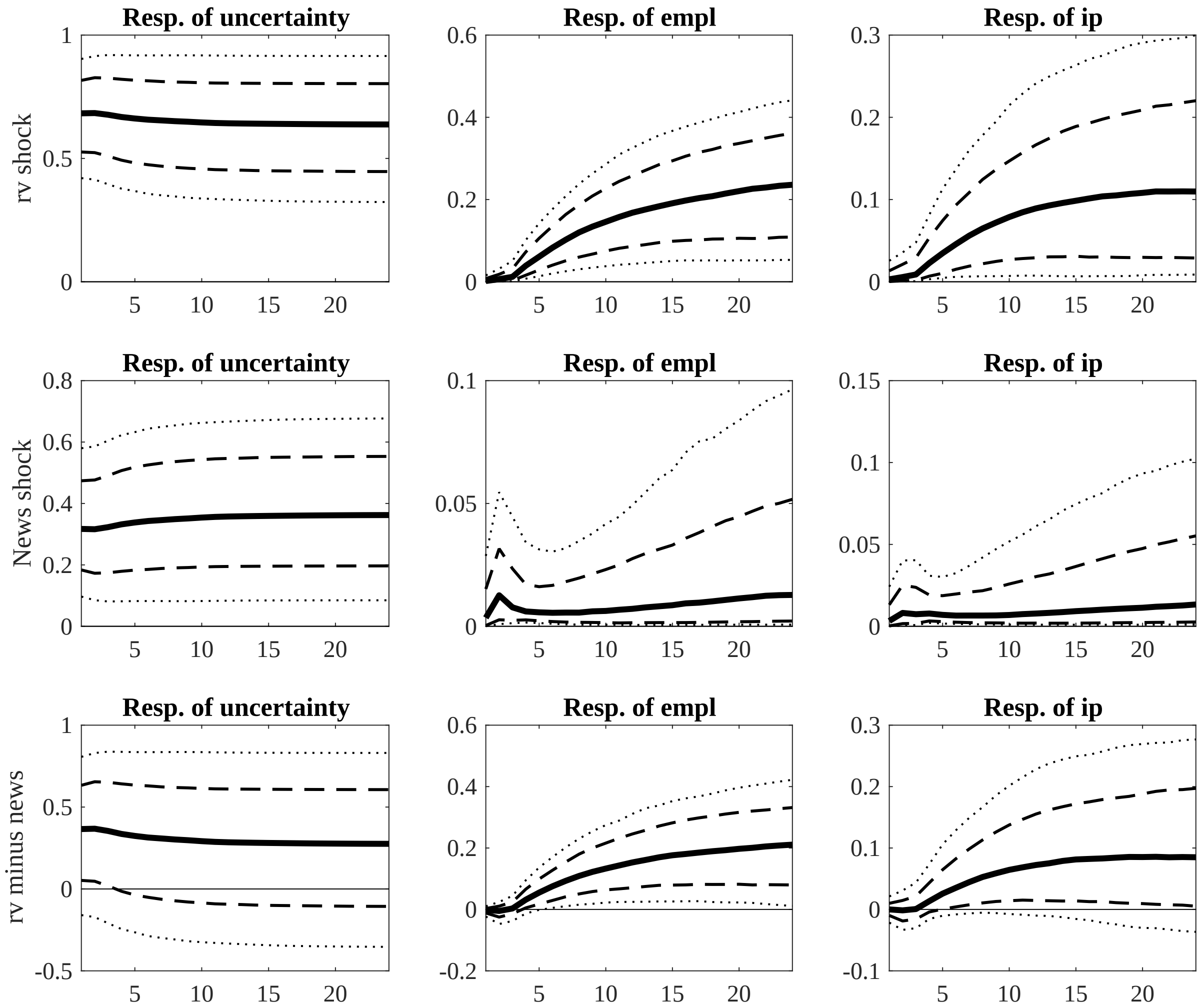
Note: Time series of realized volatility (RV), and 1-month option-implied volatility (V_1), in annualized standard deviation units. Grey bars indicate NBER recessions.

Figure 2: Impulse response functions from benchmark VAR



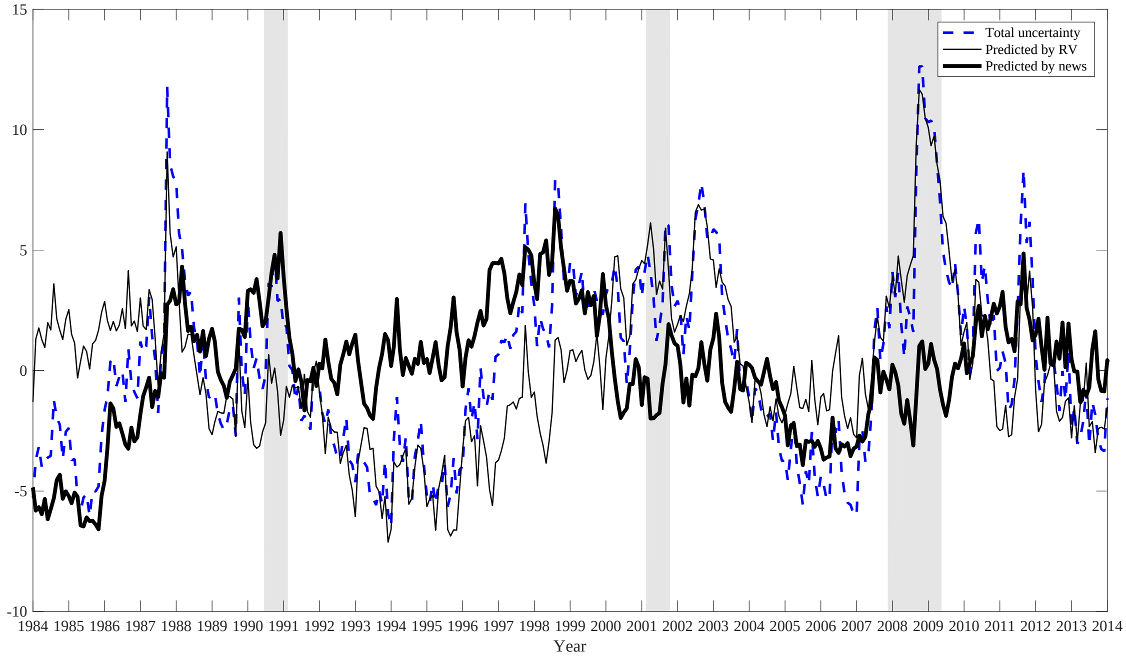
Note: Responses of rv, employment, and industrial production to shocks to rv and the identified uncertainty shock, in a VAR with rv, v_1, federal funds rate, log employment, and log industrial production. The IRFs are scaled so that the two shocks have equal cumulative effects on rv over months 2–24 following the shock. The sample period is 1983–2014. The dotted lines are 68% and 90% confidence intervals.

Figure 3: Forecast error variance decomposition

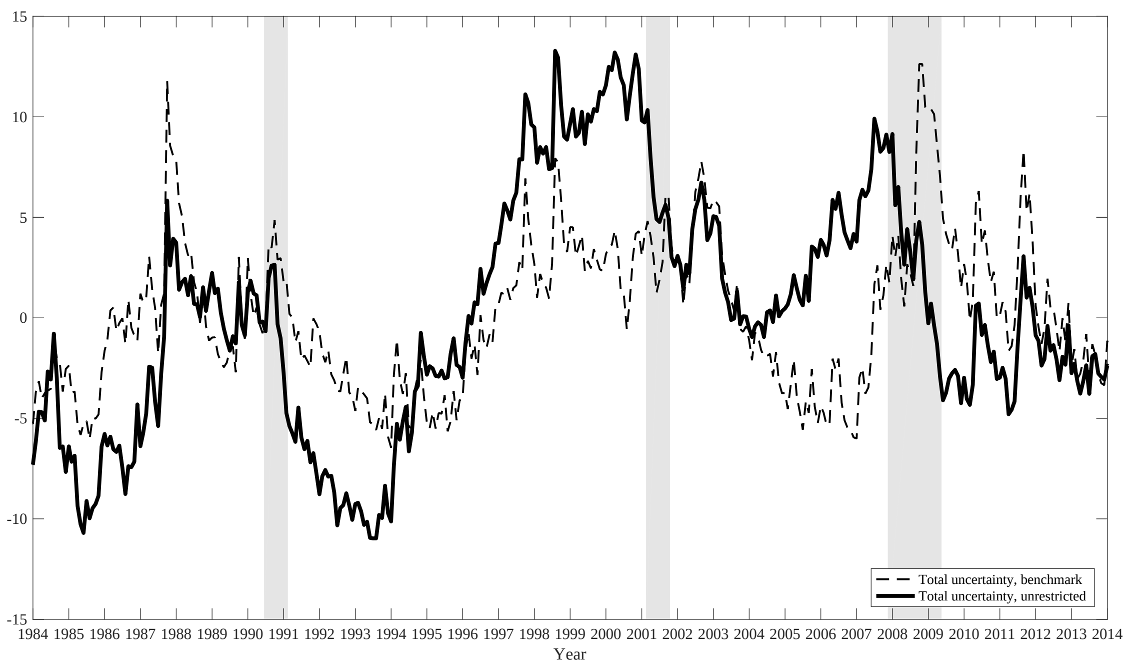


Note: Fraction of the forecast error variance of uncertainty (the expected sum of rv over the next 24 months), employment, and industrial production to shocks to rv and uncertainty in the VAR of figure 2.

Figure 4: Fitted uncertainty



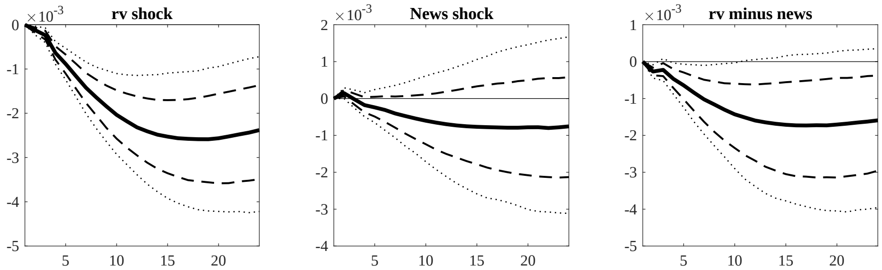
(a) Decomposition of total uncertainty in the benchmark specification



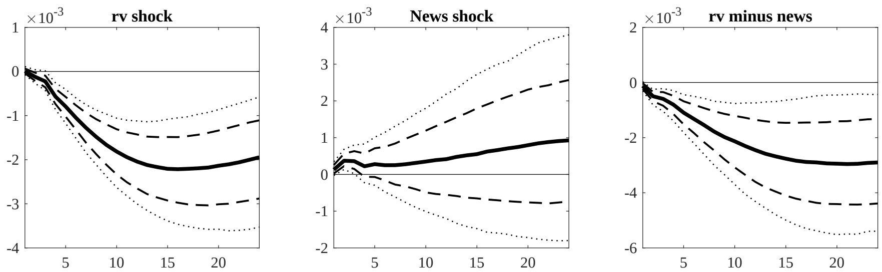
(b) Total uncertainty in benchmark vs. unrestricted specification

Note: The top panel reports a decomposition of total uncertainty (the conditional expectation of the sum of rv over the next 24 months) between the component driven by the rv shock and the component driven by the uncertainty shock in the benchmark model. The bottom panel reports the total uncertainty in the benchmark model and in the unrestricted specification.

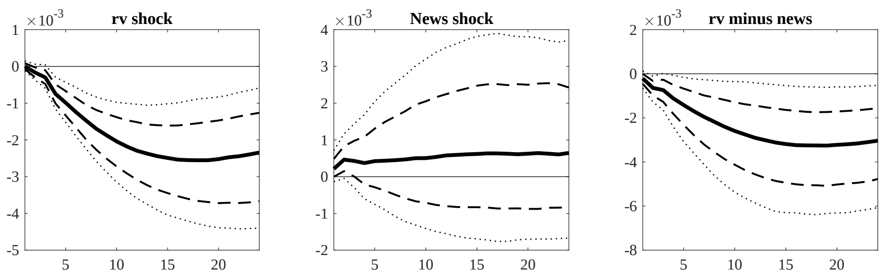
Figure 5: Response of employment to rv and uncertainty shocks across specifications



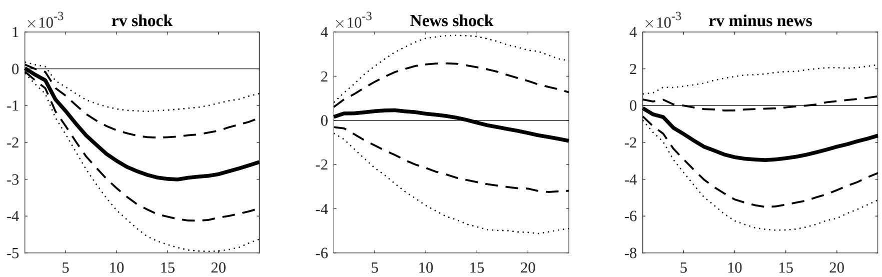
(a) rv and v_1 ordered last



(b) Replacing v_1 with v_6



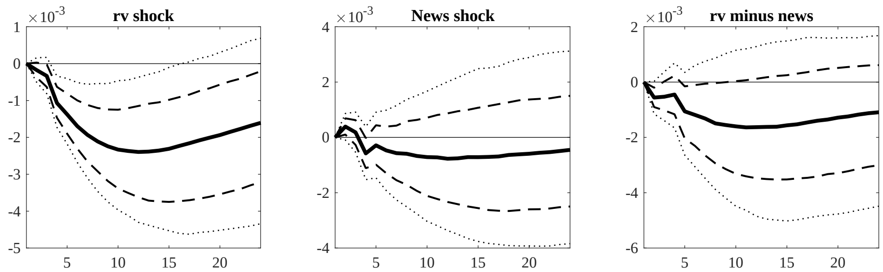
(c) Lasso



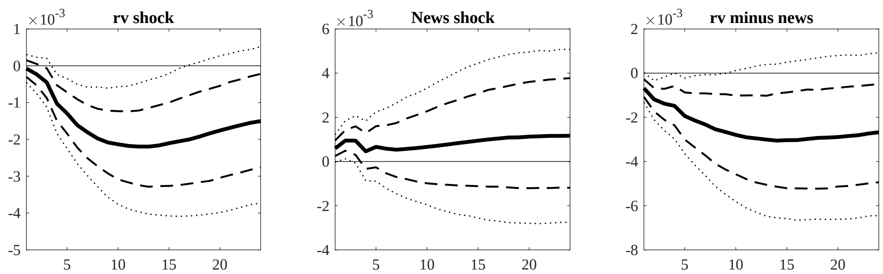
(d) Unrestricted

Note: Response of employment to RV shocks (left panels) and news shocks (middle panels) with the difference in the right panel and different model specifications in each row. Row (a) orders rv and v_1 last. Row (b) uses v_6 instead of v_1. Row (c) uses lasso to estimate the VAR (see section A.2 for details). Row (d) estimates the benchmark VAR without any coefficient restrictions.

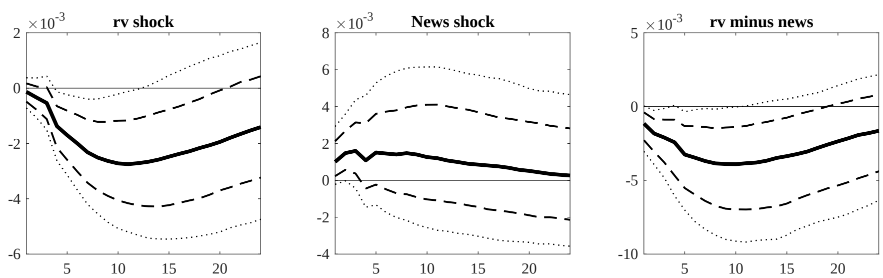
Figure 6: Response of IP to rv and uncertainty shocks across specifications



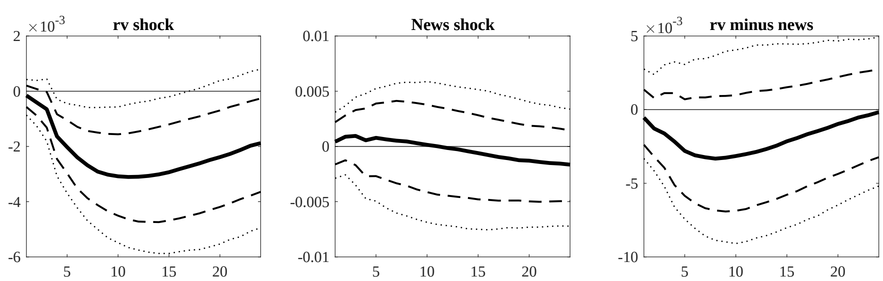
(a) rv and $v1$ ordered last



(b) Replacing v_1 with v_6



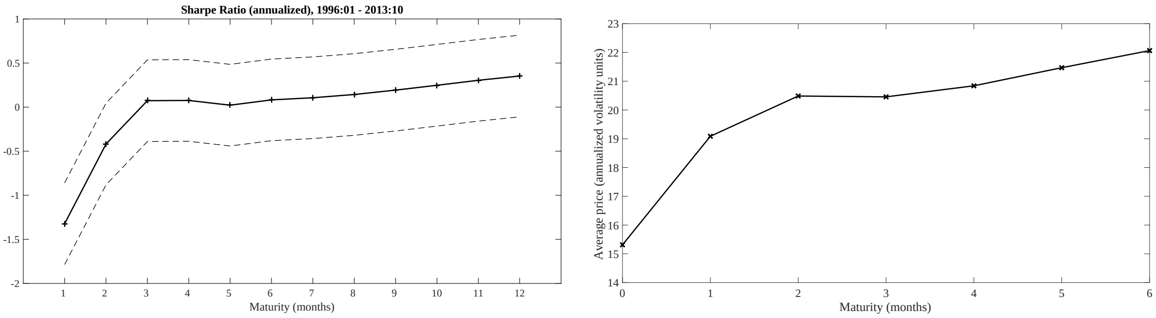
(c) Lasso



(d) Unrestricted

Note: See figure 5

Figure 7: Forward variance claims: returns and prices

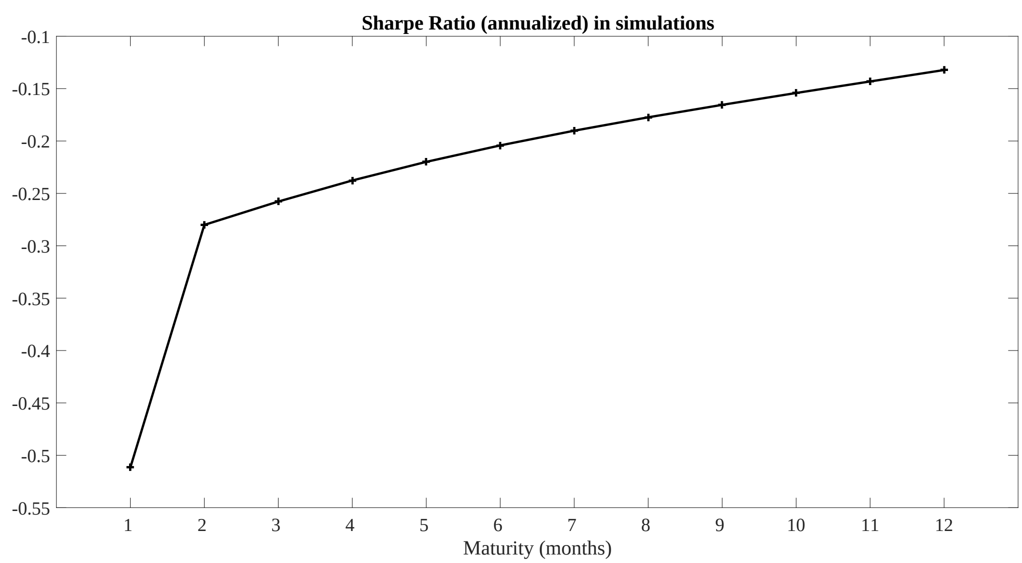


(a) Sharpe ratios

(b) Average prices

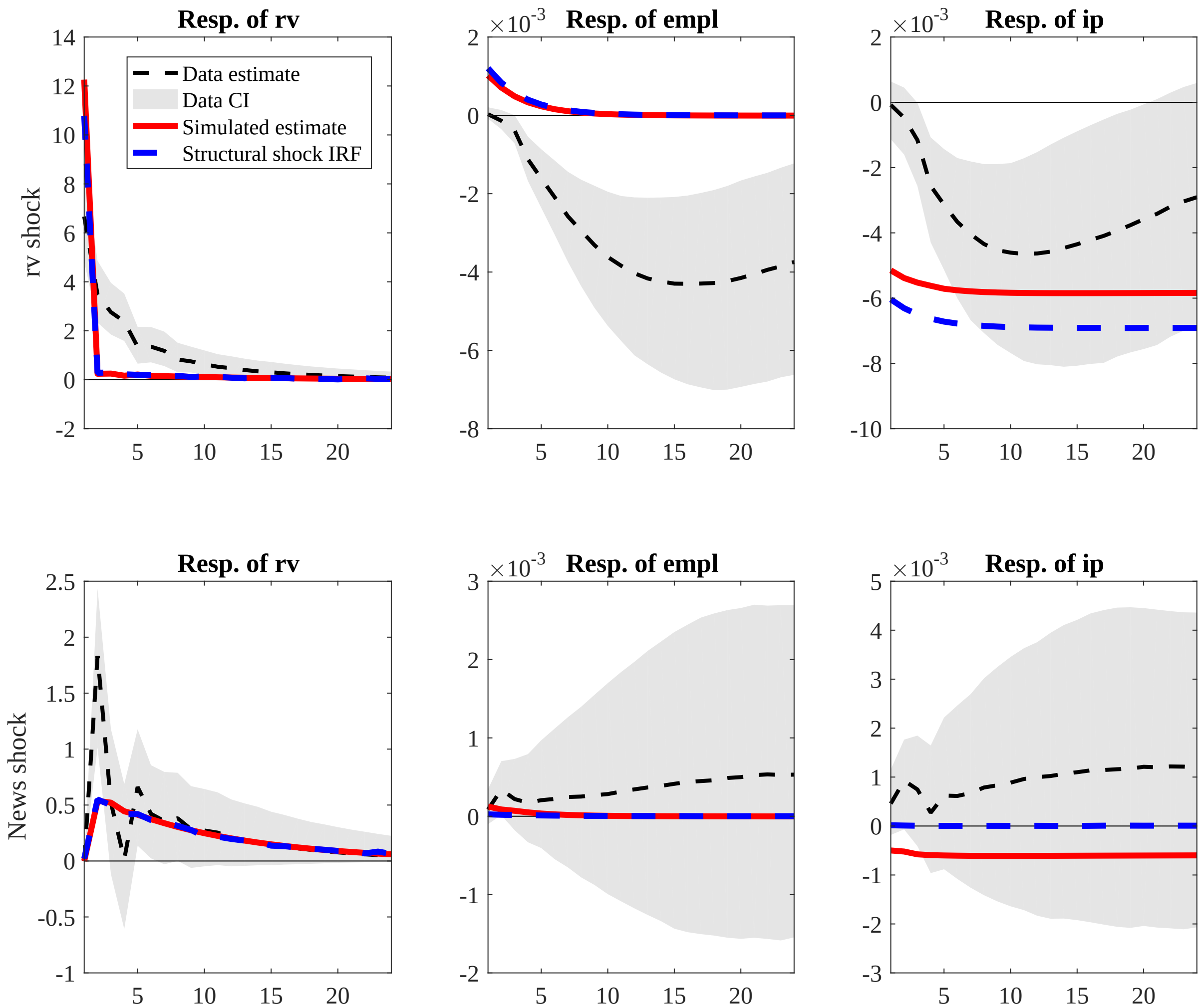
Note: Panel A shows the annualized Sharpe ratio for the forward variance claims, constructed using variance swaps. The returns are calculated assuming that the investment in an n-month variance claim is rolled over each month. Dotted lines represent 95% confidence intervals. All tests for the difference in Sharpe ratio between the 1-month variance swap and any other maturity confirm that they are statistically different with a p-value of 0.03 (for the second month) and < 0.01 (for all other maturities). The sample used is 1996–2013. For more information on the data sources, see Dew-Becker et al. (2017). Panel B shows the average prices across maturities of synthetic forward variance claims constructed from option prices for the period 1983–2014. All prices are reported in annualized standard deviation units. Maturity zero corresponds to average realized volatility.

Figure 8: Annual Sharpe ratios on forward claims (simulated structural model)



Note: Annual Sharpe ratios on forward variance claims in the simulated model of section 8. The Sharpe ratios are constructed as in Figure 7.

Figure 9: IRFs from structural model



Note: Impulse response functions from data simulated from the model in Section 8. Solid lines correspond to IRFs estimated using our VAR methodology as in Figure 2. Dashed lines correspond to IRFs for the two structural shocks $J\nu_t$ and η_t.

Table 1: Relationship between employment, industrial production and volatility

	Employment		Industrial Production	
V_1	-0.12**	0.05	-0.06*	0.12*
	(0.04)	(0.07)	(0.03)	(0.07)
RV		-0.20**		-0.21**
		(0.09)		(0.09)

Note: Results from regressions of employment and industrial production on the current value and four lags of implied and realized volatility. The coefficients and standard errors in the table are for the sum of the coefficients on each variable. Standard errors are calculated using the Newey–West (1987) method with 12 lags.

Table 2: Predictability of 6-month rv

Predictors	(1)	(2)	(3)	(4)	(5)
rv_t	0.16*	0.15	0.16*	0.14	0.09
	(0.10)	(0.09)	(0.09)	(0.09)	(0.09)
$v_{1,t}$	0.57***	0.46***	0.60**	0.56***	0.60***
	(0.14)	(0.16)	(0.29)	(0.14)	(0.15)
$v_{6,t}$			-0.03		
			(0.30)		
rv_{t-1}		0.12**			
		(0.05)			
FFR				0.01	
				(0.02)	
$\Delta empl$				-2.93	
				(10.92)	
Δip				-5.74	
				(3.52)	
PC1					-0.010
					(0.009)
PC2					-0.017
					(0.012)
PC3					-0.011
					(0.007)
$R_{S\&P}$					0.12
					(0.41)
Default spread					0.07
					(0.09)
Adj. R^2	0.44	0.45	0.44	0.45	0.46

Note: Results of linear predictive regressions of realized volatility over the next six months on lagged rv, option-implied volatility, and various macroeconomic variables, with Hansen–Hodrick (1980) standard errors using a 6-month lag window. PC1–3 are principal components from the data set used in Ludvigson and Ng (2007). The default spread is the difference in yields on Baa and Aaa bonds. The sample is 1983–2014.

Table 3: Skewness

Panel A: real economic activity	Monthly	Quarterly	Start of sample (year)
Employment	-0.41	-0.41	1948
Capacity Utilization	-1.02	-1.30	1967
IP	0.17	-0.16	1948
IP, starting 1960	-0.93	-1.28	1960
Y		-0.11	1947
C		-0.28	1947
I		-0.03	1947

Panel B: skewness of S&P 500 monthly returns	
Implied (since 1990)	-1.81
Realized (since 1926)	0.36
Realized (since 1948)	-0.42
Realized (since 1990)	-0.61

Note: Panel A reports the skewness of changes of employment, capacity utilization, industrial production (beginning both in 1948 and in 1960), GDP, consumption and investments. The first column reports the skewness of monthly changes, the second column the skewness of quarterly changes. Panel B reports the realized skewness of S&P 500 monthly returns in different periods, as well as the implied skewness computed by the CBOE using option prices.

Table 4: Model Calibration

Panel A: Moments	Model			Data		
	Mean	Std.	Skewness	Mean	Std.	Skewness
Returns	6.50	15.00	-4.92	7.46	14.77	-0.48
Output	1.99	2.02	-4.48	1.33	1.92	-0.11
Investment	1.99	3.64	-4.46	2.35	7.43	-0.03
Consumption	1.99	1.58	-3.41	1.20	1.06	-0.28

Panel B: Corr. of VAR and structural shocks		Structural shocks		
		Jv_t	η_t	ε_t
	RV	0.96	0.01	0.00
VAR identified shocks	Uncertainty	-0.01	0.91	-0.14

Note: Panel A reports the mean, standard deviation, and skewness of financial and macroeconomic variables in the data and in the model. Panel B shows the correlation between the structural shocks in the model and the shocks identified in the VAR.

A.1 Construction of option-implied volatility, V_n

In this section we describe the details of the procedure we use to construct model implied uncertainty at different horizons, starting from our dataset of end-of-day prices for American options on S&P 500 futures from the CME.

Our implied volatility is written as a function of option prices,

$$
\begin{aligned}
V_{n,t} &\equiv Var_t^Q\left[s_{t+n}\right] && \text{(A.1)}\\
&= 2\int_0^\infty \frac{1-\log\left(\frac{K}{e^{rt}S_t}\right)}{B_t\left(n\right)K^2}O\left(K\right)dK - \left(e^{rt}\int_0^\infty \frac{O\left(K\right)}{B_t\left(n\right)K^2}dK\right)^2 && \text{(A.2)}
\end{aligned}
$$

Note that this formula holds generally, requiring only the existence of a well-behaved pricing measure; there is no need to assume a particular specification for the returns process. $Var_t^Q\left[s_{t+n}\right]$ is calculated as an integral over option prices, where K denotes strikes, $O_t\left(n,K\right)$ is the price of an out-of-the-money option with strike K and maturity n, and $B_t\left(n\right)$ is the price at time t of a bond paying one dollar at time $t+n$. $V_{n,t}$ is equal to the option-implied variance of log stock prices n months in the future.

The result for $Var_t^Q\left[s_{t+n}\right]$ is obtained from equation 3 in Bakshi, Kapadia, and Madan (2003) by first setting $H\left(S\right)=\log\left(S\right)$ to obtain $E_t^Q\left[\log S_{t+n}\right]$ and then defining $G\left(S\right)=\left(\log\left(S\right)-E_t^Q\left[\log S_{t+n}\right]\right)^2$ and inserting it into equation 3 in place of H.

A.1.1 Main steps of construction of V_n

A first step in constructing the model-free implied volatility is to obtain implied volatilities corresponding to the observed option prices. We do so using a binomial model.[1] For the most recent years, CME itself provides the implied volatility together with the option price. For this part of the sample, the IV we estimate with the binomial model and the CME's IV have a correlation of 99%, which provides an external validation on our implementation of the binomial model.

Once we have estimated these implied volatilities, we could in theory simply invert them to yield implied prices of European options on forwards. These can then be used to compute V_n directly as described in equation (10).

In practice, however, an extra step is required before inverting for the European option prices and integrating to obtain the model-free implied volatility. The model-free implied volatility defined in equation (10) depends on the integral of option prices over *all strikes*, but option prices are only observed at discrete strikes. We are therefore forced to interpolate option prices between available strikes and also extrapolate beyond the bounds of observed strikes.[2] Following the literature, we fit a parametric model to the Black–Scholes implied volatilities of the options and use the model to then interpolate and extrapolate across all strikes (see, for example, Jiang and Tian (2007), Carr

[1] See for example Broadie and Detemple (1996) and Bakshi, Kapadia, and Madan (2003), among others.

[2] See Jiang and Tian (2007) for a discussion of biases arising from the failure to interpolate and extrapolate.

and Wu (2009), Taylor, Yadav, and Zhang (2010), and references therein). Only after this extra interpolation-extrapolation step, the fitted implied volatilities are then inverted to yield option prices and compute V_n according to equation (10). To interpolate and extrapolate the implied volatility curve, we use the SVI (stochastic volatility inspired) model of Gatheral and Jacquier (2014).

In the next sections, we describe in more detail the interpolation-extrapolation step of the procedure (SVI fitting) as well as our construction of V_n after fitting the SVI curve. Finally, we report a description of the data we use and some examples and diagnostics on the SVI fitting method.

A.1.2 SVI interpolation: theory

There are numerous methods for fitting implied volatilities across strikes. Homescu (2011) provides a thorough review. We obtained the most success using Gatheral's SVI model (see Gatheral and Jacquier 2014). SVI is widely used in financial institutions because it is parsimonious but also known to approximate well the behavior of implied volatility in fully specified option pricing models (e.g. Gatheral and Jacquier (2011)); SVI also satisfies the limiting results for implied volatilities at very high and low strikes in Lee (2004), and, importantly, ensures that no-arbitrage conditions are not violated.

The SVI model simply assumes a hyperbolic relationship between implied variance (the square of the Black–Scholes implied volatility) and the log moneyness of the option, k (log strike/forward price).

$$\sigma_{BS}^2\left(k\right) = a + b\left(\rho\left(k-m\right) + \sqrt{\left(k-m\right)^2 + \sigma^2}\right)$$

where $\sigma_{BS}^2\left(k\right)$ is the implied variance under the Black–Scholes model at log moneyness k. SVI has five parameters: a, b, ρ, m, and σ. The parameter ρ controls asymmetry in the variances across strikes. Because the behavior of options at high strikes has minimal impact on the calculation of model-free implied volatilities, and because we generally observe few strikes far above the spot, we set $\rho = 0$ (in simulations with calculating the VIX for the S&P 500 – for which we observe a wide range of options – we have found that including or excluding ρ has minimal impact on the result).

We fit the parameters of SVI by minimizing the sum of squared fitting errors for the observed implied volatilities. Because the fitted values are non-linear in the parameters, the optimization must be performed numerically. We follow the methodology in Zeliade (2009) to analytically concentrate a and b out of the optimization. We then only need to optimize numerically over σ and m (as mentioned above, we set $\rho = 0$). We optimize with a grid search over $\sigma \times m = [0.001, 10] \times [-1, 1]$ followed by the simplex algorithm.

For many date/firm/maturity triplets, we do not have a sufficient number of contract observations to fit the implied volatility curve (i.e. sometimes fewer than four). We therefore include strike/implied volatility data from the two neighboring maturities and dates in the estimation. The parameters of SVI are obtained by minimizing squared fitting errors. We reweight the observations from the neighboring dates and maturities so that they carry the same amount of weight as the observations from the date and maturity of interest. Adding data in this way encourages smoothness in the estimates over time and across maturities but it does not induce a systematic upward or downward bias. We drop all date/firm/maturity triplets for which we have fewer than four total options with $k < 0$ or fewer than two options at the actual date/firm/maturity (i.e. ignoring the data from the neighboring dates and maturities).

When we estimate the parameters of the SVI model, we impose conditions that guarantee the absence of arbitrage. In particular, we assume that $b \leq \frac{4}{(1+|\rho|)T}$, which when we assume $\rho = 0$, simplifies to $b \leq \frac{4}{T}$. We also assume that $\sigma > 0.0001$ in order to ensure that the estimation is well defined. Those conditions do not necessarily guarantee, though, that the integral determining the model-free implied volatility is convergent (the absence of arbitrage implies that a risk-neutral probability density exists – it does not guarantee that it has a finite variance). We therefore eliminate observations where the integral determining the model-free implied volatility fails to converge numerically. Specifically, we eliminate observations where the argument of the integral does not approach zero as the log strike rises above two standard deviations from the spot or falls more than five standard deviations below the strike (measured based on the at-the-money implied volatility).

A.1.3 Construction of V_n from the SVI fitted curve

After fitting the SVI curve for each date and maturity, we compute the integral in equation (10) numerically, over a range of strikes from -5 to +2 standard deviations away from the spot price.[3] We then have V_n for every firm/date/maturity observation. The model-free implied volatilities are then interpolated (but not extrapolated) to construct V_n at maturities from 1–6 months for each firm/date pair.

A.1.4 Data description and diagnostics of SVI fitting

Our dataset consists of 2.3 million end-of-day prices for all American options on S&P 500 futures from the CME.

When more than one option (e.g. a call and a put) is available at any strike, we compute IV at that strike as the average of the observed IVs. We keep only IVs greater than zero, at maturities higher than 9 days and lower than 2 years, for a total of 1.9 million IVs. The number of available options has increased over time, as demonstrated by Figure A.2 (top panel), which plots the number of options available for V_n estimation in each year.

The maturity structure of observed options has also expanded over time, with options being introduced at higher maturities and for more intermediate maturities. Figure A.1 (top panel)

[3] In general this range of strikes is sufficient to calculate V_n. However, the model-free implied volatility technically involves an integral over the entire positive real line. Our calculation is thus literally a calculation of Andersen and Bondarenko's (2007) corridor implied volatility. We use this fact also when calculating realized volatility.

reports the cross-sectional distribution of available maturities in each year to estimate the term structure of the model-free implied volatility. The average maturity of available options over our sample was 4 months, and was relatively stable. The maximum maturity observed ranged from 9 to 24 months and varied substantially over time.

Crucial to compute the model-free implied volatility is the availability of IVs at low strikes, since options with low strikes receive a high weight in the construction of V_n. The bottom panel of Figure A.1 reports the minimum observed strike year by year, in standard deviations below the spot price. In particular, for each day we computed the minimum available strike price, and the figure plots the average of these minimum strike price across all days in each year; this ensures that the number reported does not simply reflect outlier strikes that only appear for small parts of each year.

Figure A.1 shows that in the early part of our sample, we can typically observe options with strikes around 2 standard deviations below the spot price; this number increases to around 2.5 towards the end of the sample.

These figures show that while the number of options was significantly smaller at the beginning of the sample (1983), the maturities observed and the strikes observed did not change dramatically over time.

Figure A.3 shows an example of the SVI fitting procedure for a specific day in the early part of our sample (November 7th 1985). Each panel in the figure corresponds to a different maturity. On that day, we observe options at three different maturities, of approximately 1, 4, and 8 months. In each panel, the x's represent observed IVs at different values of log moneyness k. The line is the fitted SVI curve, that shows both the interpolation and the extrapolation obtained from the model.

Figure A.4 repeats the exercise in the later part of our sample (Nov. 1st 2006), where many more maturities and strikes are available.

Both figures show that the SVI model fits the observed variances extremely well. The bottom panel of Figure A.2 shows the average relative pricing error for the SVI model in absolute value. The graph shows that the typical pricing error for most of the sample is around 0.02, meaning that the SVI deviates from the observed IV by around 2% on average. Only in the very first years (up to 1985) pricing errors are larger, but still only around 10% of the observed IV.

Overall, the evidence in this section shows that our observed option sample since 1983 has been relatively stable along the main dimensions that matter for our analysis – maturity structure, strikes observed, and goodness of fit of the SVI model.

A.2 Lasso

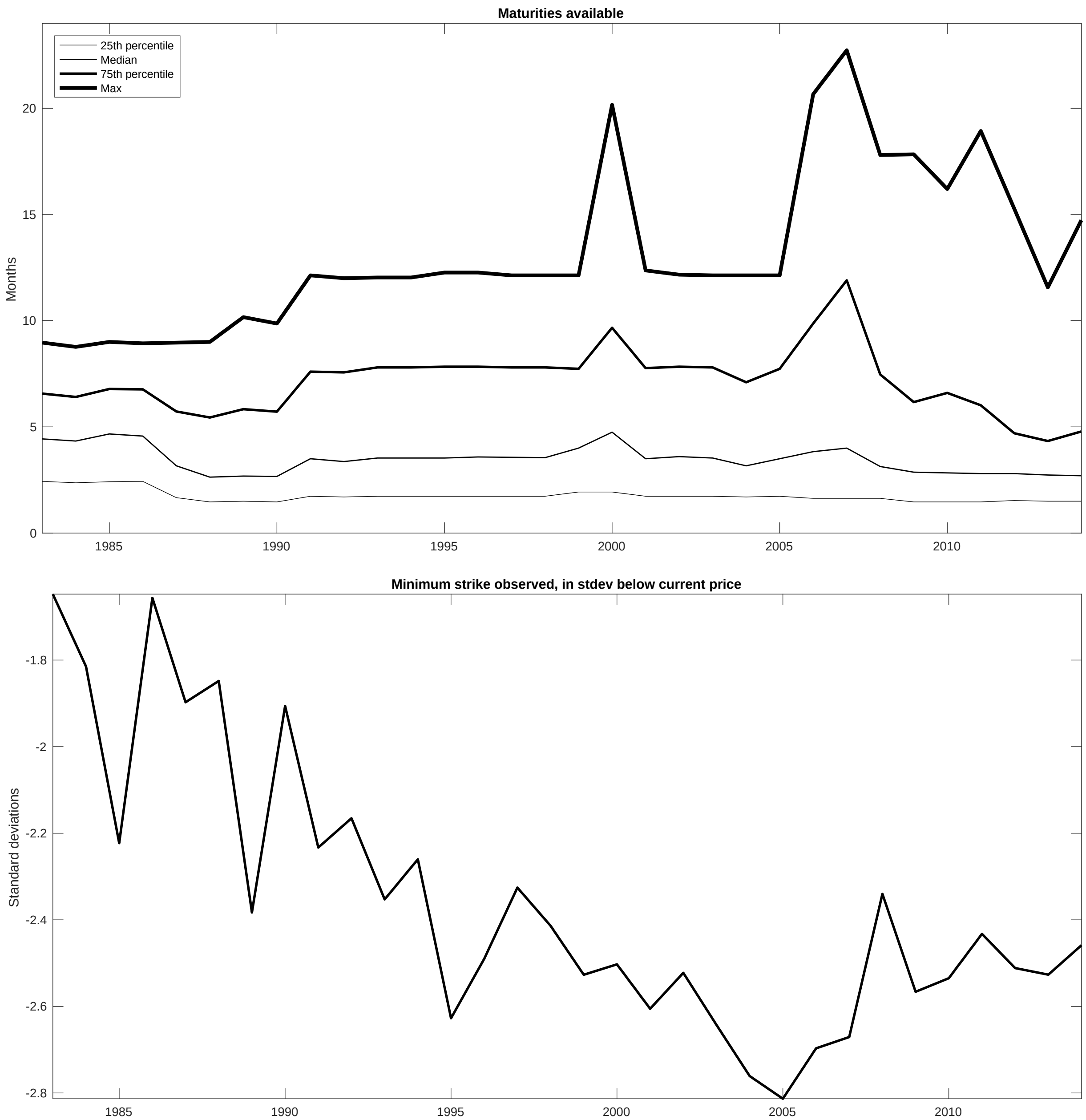
Lasso is a regularization method for regressions that penalizes coefficients based on their absolute values. Specifically, the objective that is minimized under lasso is the sum of squared residuals plus a tuning parameter, which we denote λ, multiplied by the sum of the absolute values of the coefficients.

Lasso is not invariant to the scaling of the variables in the regression. We therefore rescale the variables as follows. rv, v_6, and *slope* are all translated into z-scores. The three macro variables (FFR, *emp*, and *ip*) are multiplied by constants so that their first differences have unit variances. We use that transformation because those three variables have approximate unit roots in our sample.

We examine two methods to select λ. The first is to use leave-one-out cross validation. We choose λ separately for the three volatility and three macro series. The cross-validation criterion implies setting $\lambda = 0.013$ for the volatility series and $\lambda = 0$ (i.e. no lasso) for the macro series. the results reported in the text use this choice of λ.

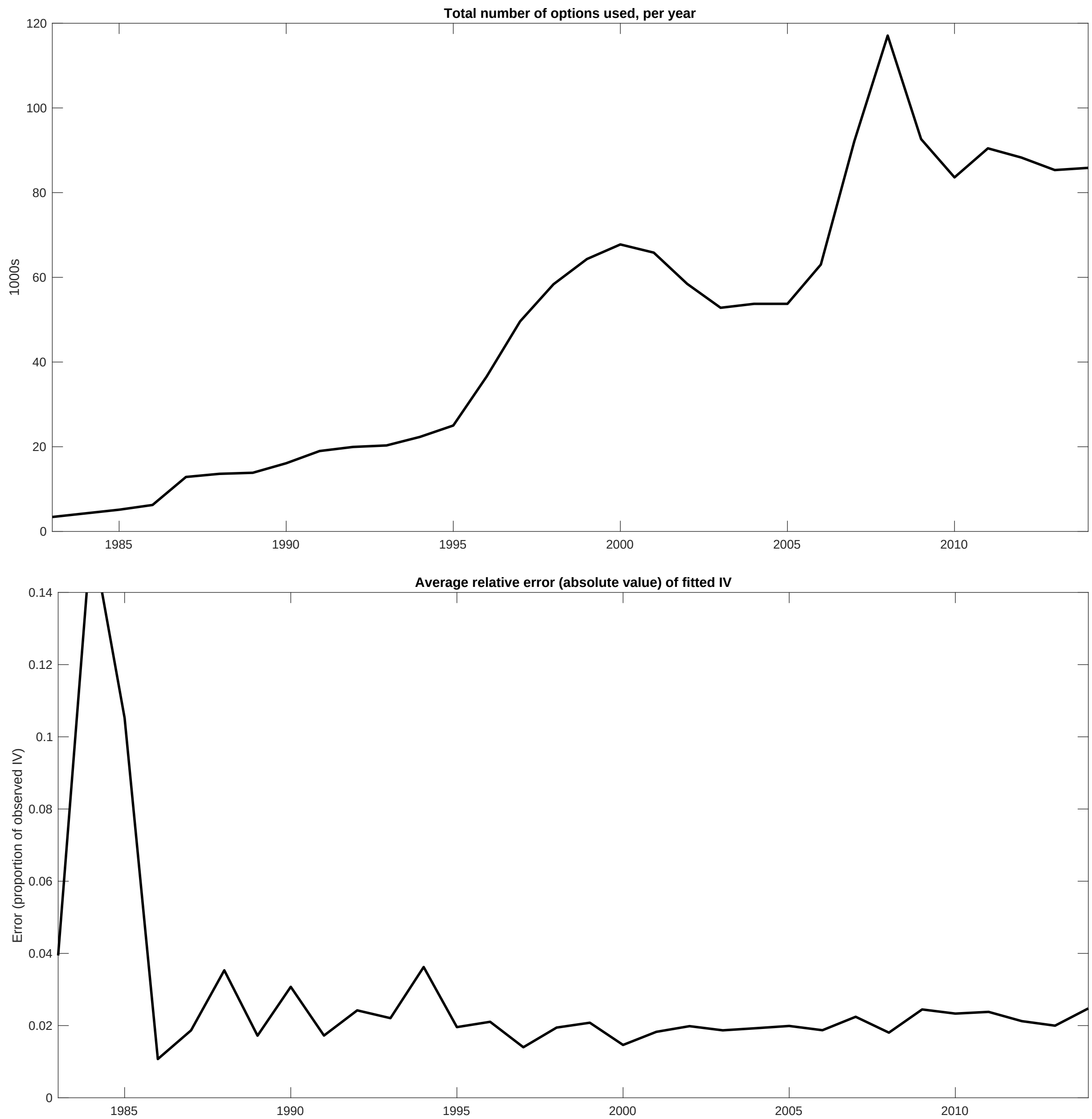
The second method is to choose the smallest (i.e. least restrictive) value of λ that causes the coefficients on all the lags of the macro variables in the rv equation to be zero, consistent with the benchmark specification. The motivation for this method is that it takes the restrictions that we impose on economic grounds and then essentially tries to impose similar restrictions on the other equations, for the sake of parity. In this case we find a value of λ of 0.055. The results with this value are not reported here but are consistent with out main findings. In this case we find slightly negative effects for uncertainty shocks, but they are still statistically significantly less negative than those for rv shocks, and the forecast error variance decompositions put an upper bound on the fraction driven by uncertainty shocks of 15 percent.

Figure A.1: Maturities and strikes in the CME dataset



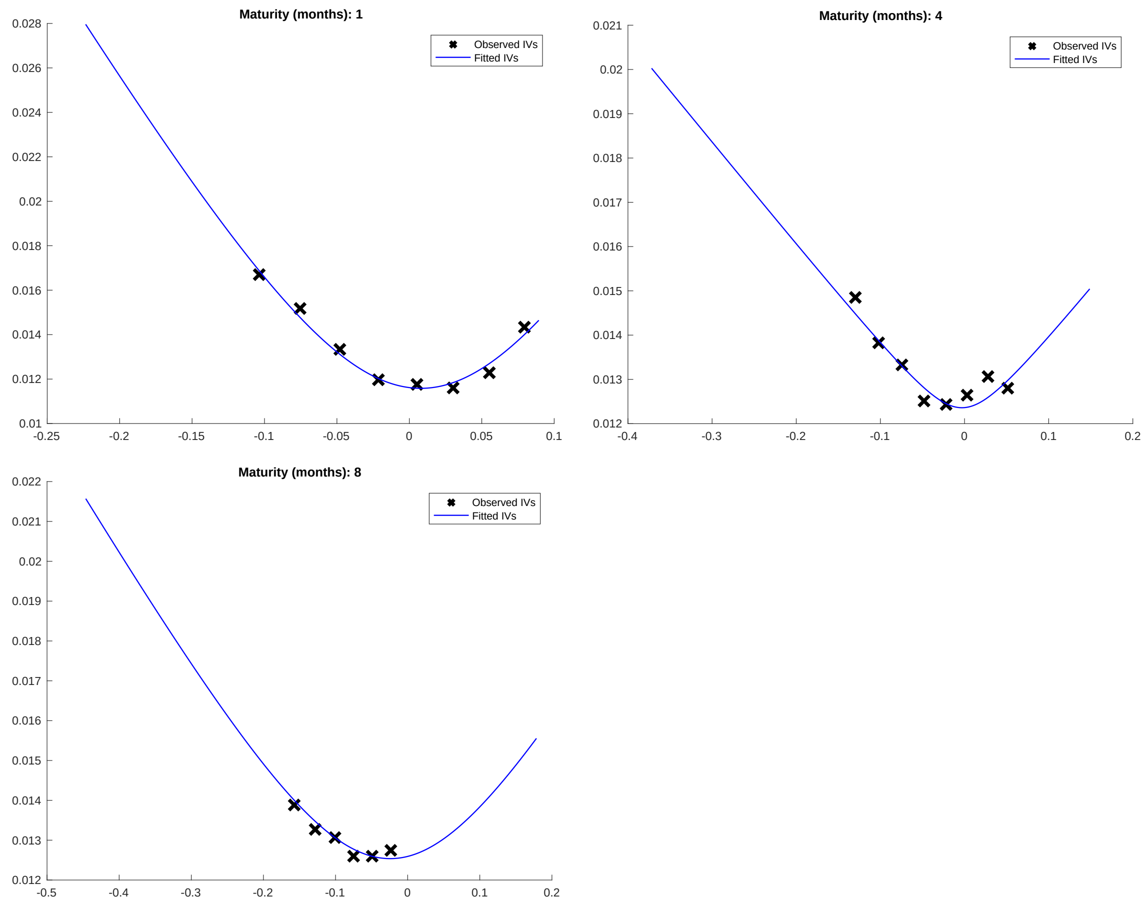
Note: The top panel reports the distribution of maturities of options used to compute implied volatility in each year, in months. The bottom panel reports the average minimum strike in each year, in standard deviations below the forward price. The number is obtained by computing the minimum observed strike in each date and at each maturity (in standard deviations below the forward price), and then averaging it within each year to minimize the effect of outliers.

Figure A.2: Number of options to construct implied volatility and pricing errors



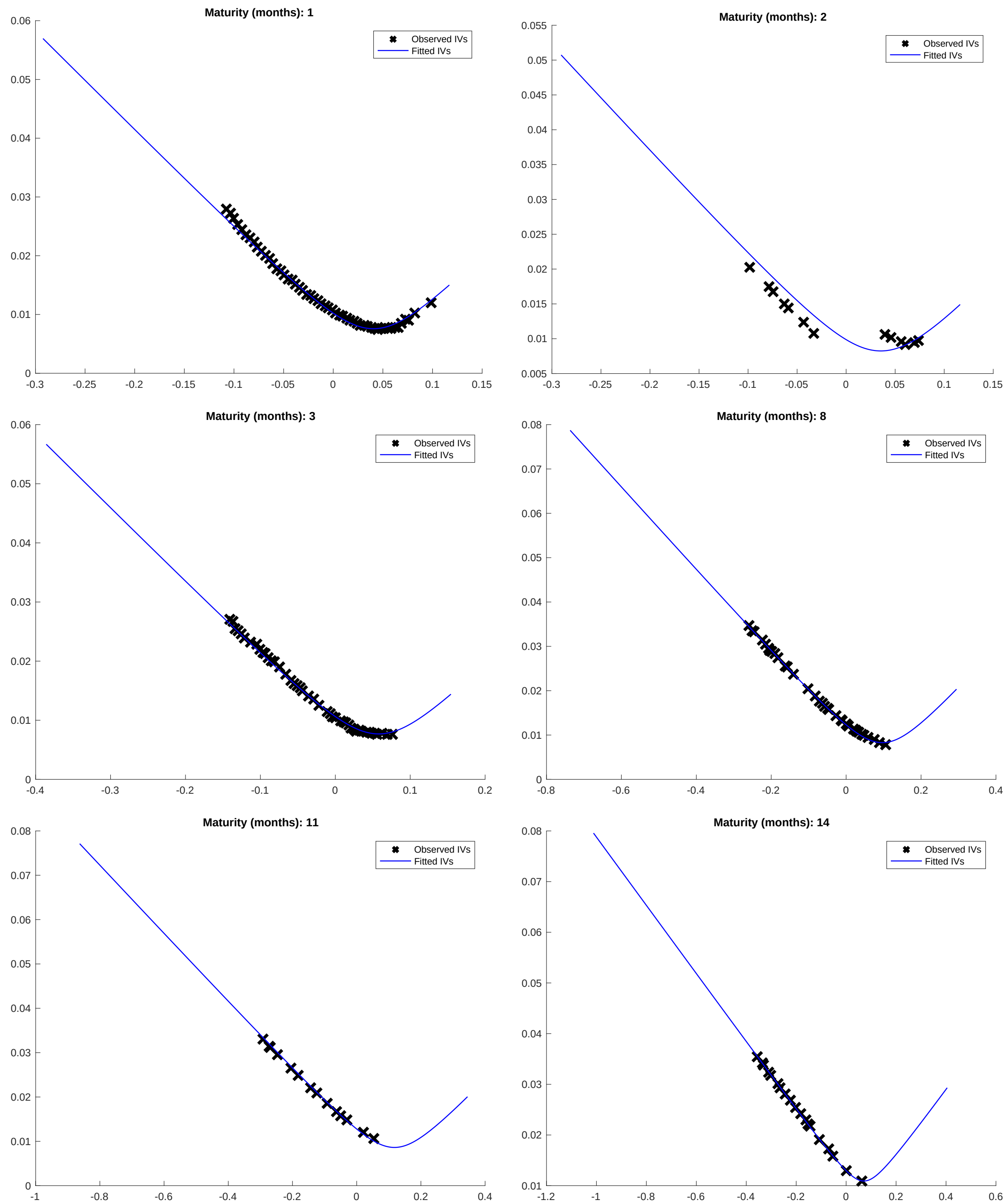
Note: The top panel reports the number of options used to compute implied volatility in each year, in thousands. The bottom panel reports the average absolute value of the pricing error of the SVI fitted line relative to the observed implied variances, in proportional terms (i.e. 0.02 means absolute value of the pricing error is 2% of the observed implied variance).

Figure A.3: SVI fit: 11/7/1985



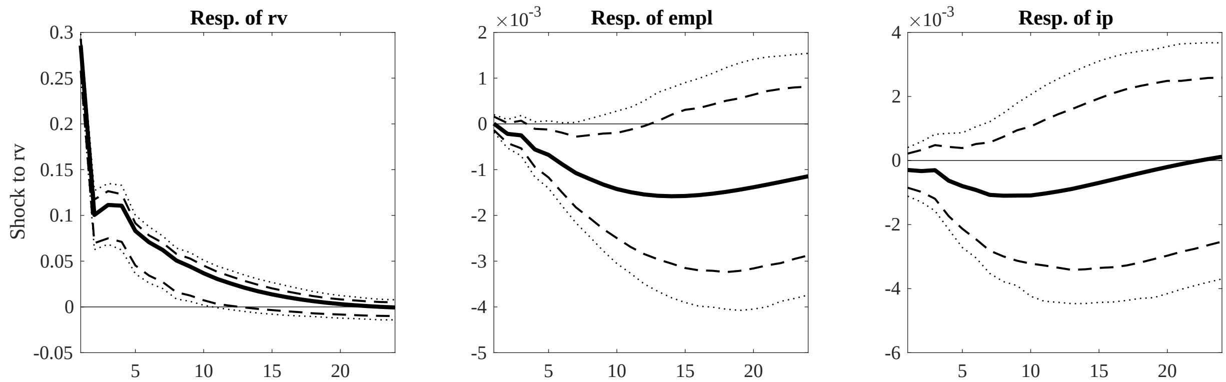
Note: Fitted implied variance curve on 11/7/1987, for the three available maturities. X axis is the difference in log strike and log forward price. x's correspond to the observed implied variances, and the line is the fitted SVI curve.

Figure A.4: SVI fit: 11/1/2006



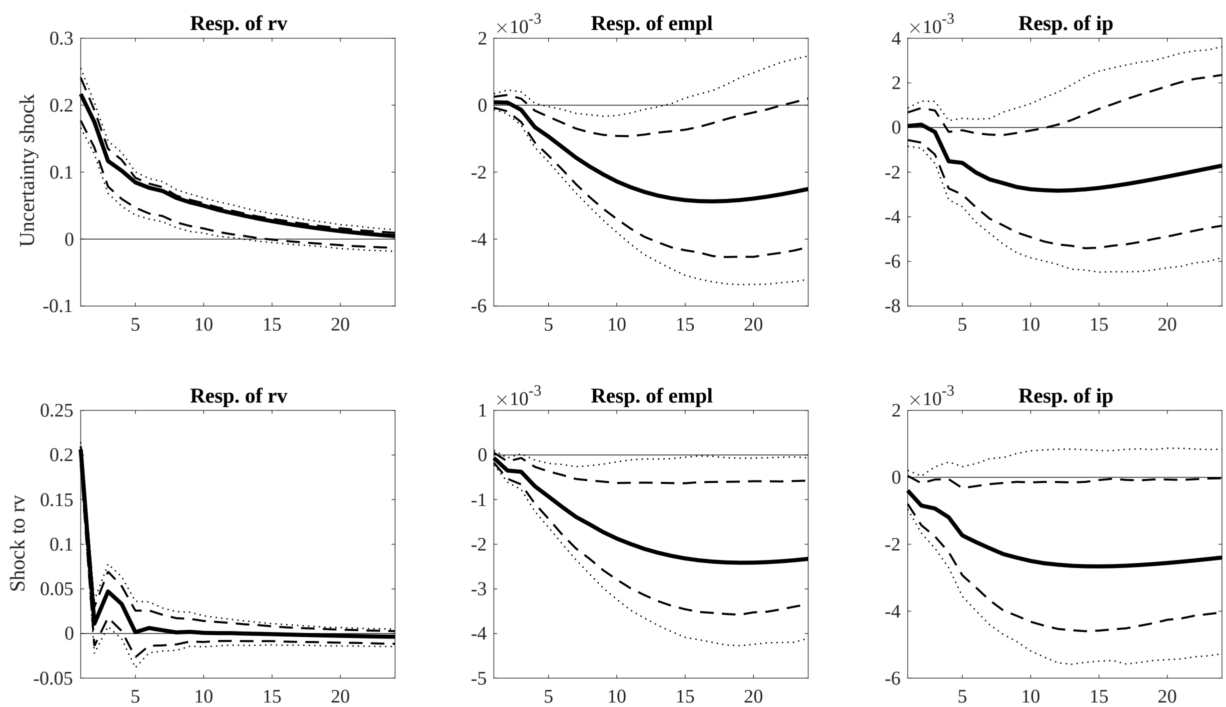
Note: Fitted implied variance curve on 11/1/2006, for the three available maturities. X axis is the difference in log strike and log forward price. x's correspond to the observed implied variances, and the line is the fitted SVI curve. On 11/1/2006 also a maturity of 5 months was available (not plotted for reasons of space).

Figure A.5: Impulse response functions from VAR with v_1 but not rv



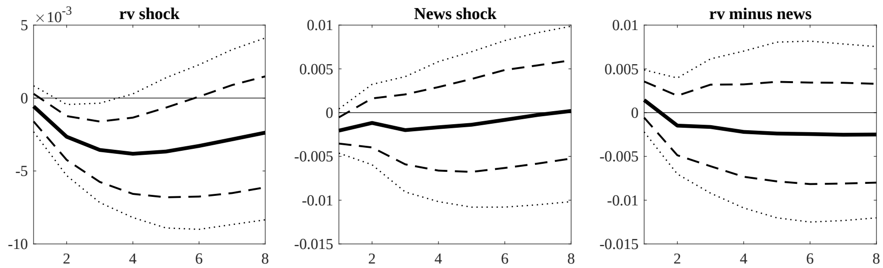
Note: The figure shows responses of volatility (measured by v_1), log employment, and log industrial production to a reduced-form shock to v_1 in a VAR with v_1, the Fed funds rate, log employment, and log industrial production with 68% and 90% confidence intervals. Sample period 1986-2014.

Figure A.6: Impulse response functions from VAR ordering uncertainty first and rv second

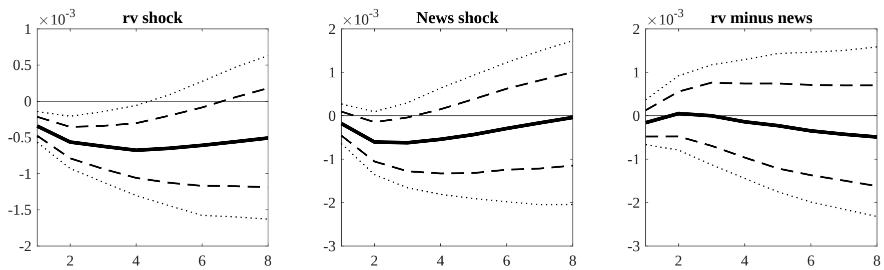


Note: See figure 2. Unlike in the baseline identification, the identified uncertainty shock is not orthogonalized with respect to rv. The rv shock in this case is the remaining part of reduced-form innovation to rv that is not spanned by the uncertainty shock.

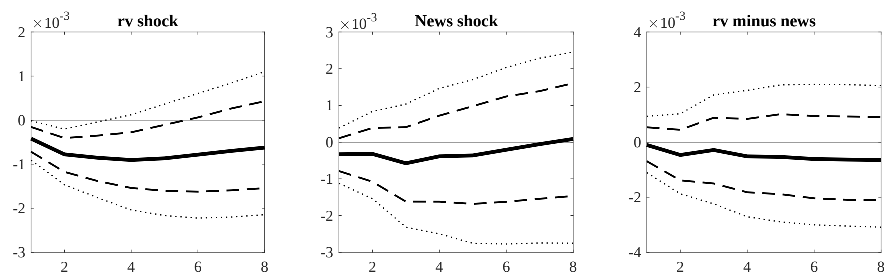
Figure A.7: Robustness: data from Basu and Bundick (2016), using v_1



(a) Investment



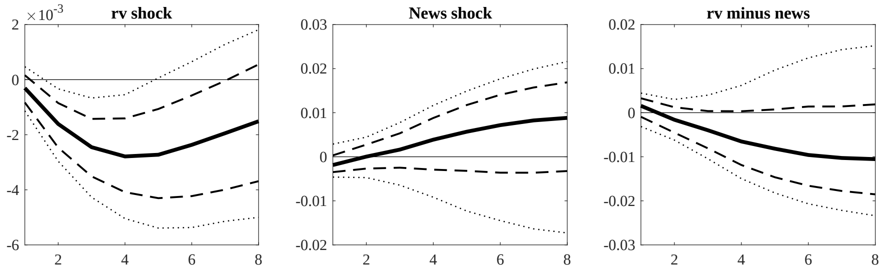
(b) Consumption



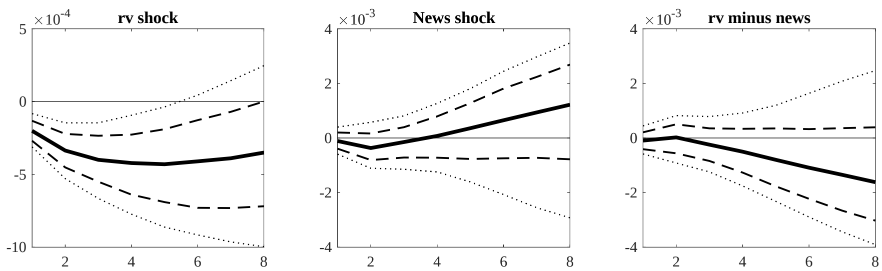
(c) GDP

Note: See figure 2. Here we use the quarterly data from Basu and Bundick (2016) as the macro time series.

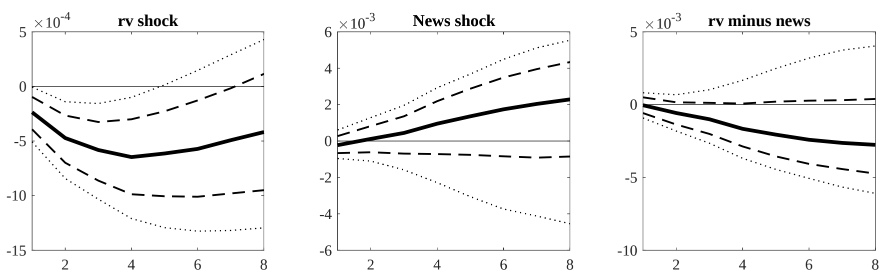
Figure A.8: Robustness: data from Basu and Bundick (2016), using v_6



(a) Investment



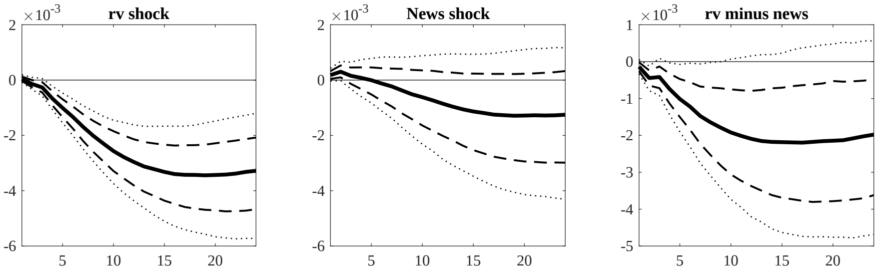
(b) Consumption



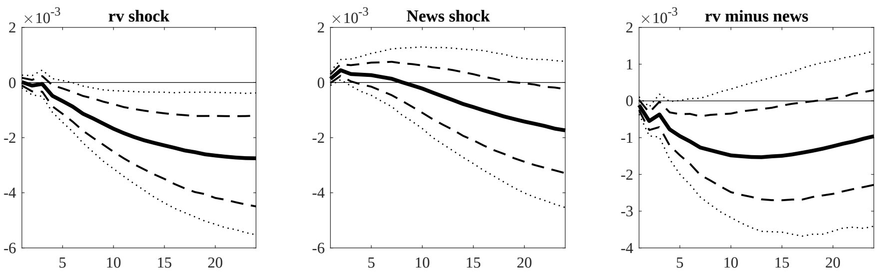
(c) GDP

Note: See figure 2. Here we use the quarterly data from Basu and Bundick (2016) as the macro time series. We use v_6 instead of v_1.

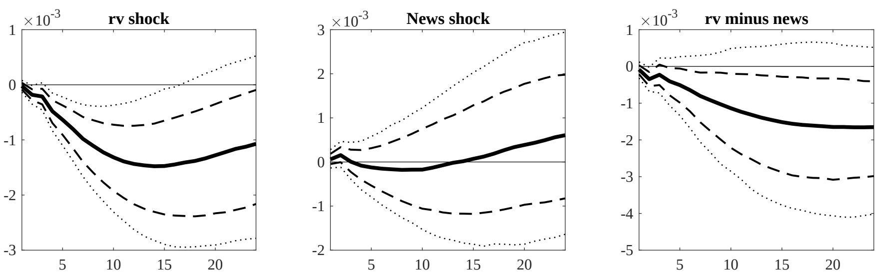
Figure A.9: Robustness: response of Employment to rv and uncertainty shocks across specifications



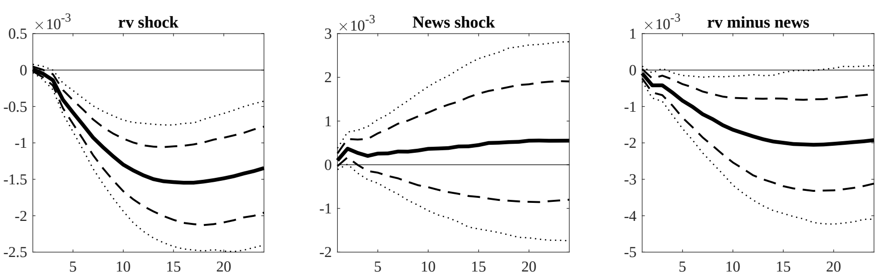
(a) Using the CBOE VIX instead of v_1



(b) Subperiod 1988-2006 (excluding 1987 crash and financial crisis)



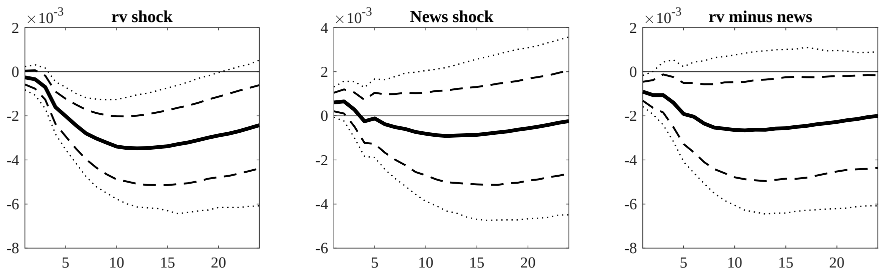
(c) Adding the S&P 500 level as first shock



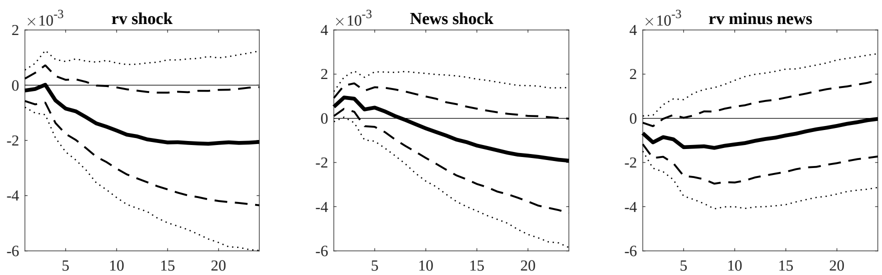
(d) Using RV and V_1 in levels, not logs

Note: Response of employment to RV shocks (left panels) and uncertainty (middle panels) with the difference in the right panel and different model specifications in each row. Row (a) detrends the macroeconomic time series via HP filter. Row (b) estimates the VAR in the subsample 1988-2006, which excludes both RV peaks (1987 crash and financial crisis). Row (c) orthogonalizes both the rv and the uncertainty shocks with respect to the reduced-form innovation in the S&P 500, as in Bloom (2009). Row (d) uses RV and V_1 in levels, not logs.

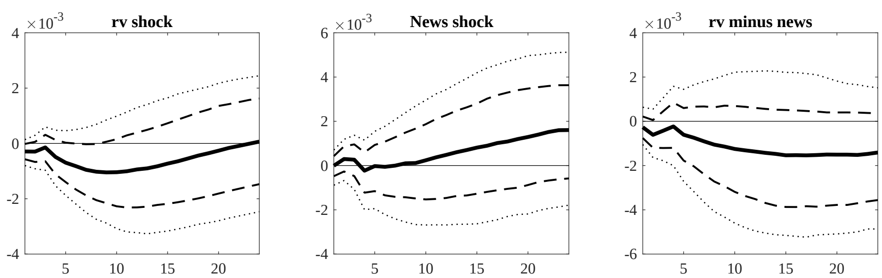
Figure A.10: Robustness: response of IP to rv and uncertainty shocks across specifications



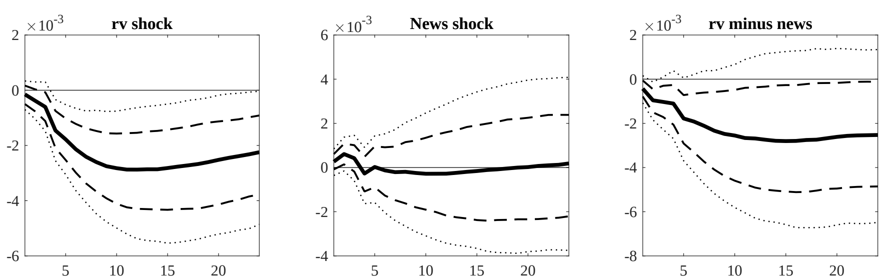
(a) Using the CBOE VIX instead of v_1



(b) Subperiod 1988-2006 (excluding 1987 crash and financial crisis)



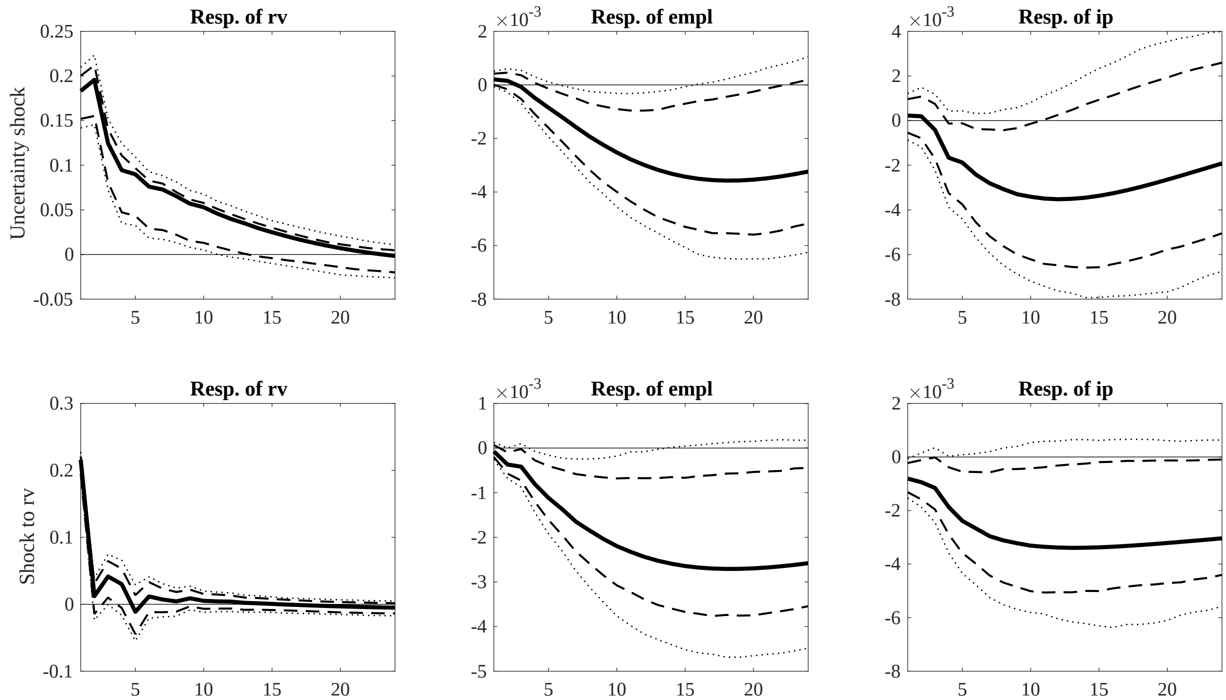
(c) Adding the S&P 500 level as first shock



(d) Using RV and V_1 in levels, not logs

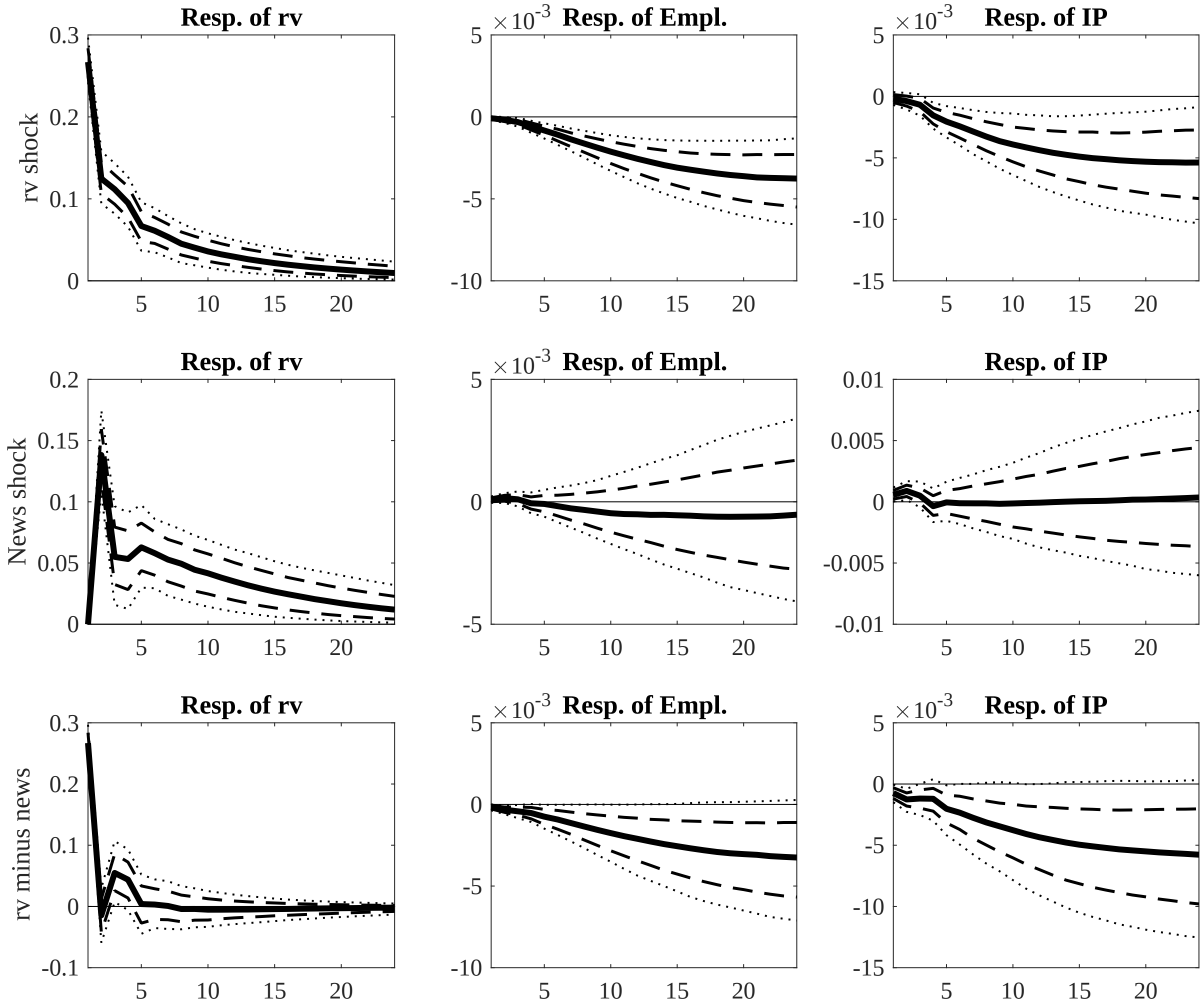
Note: See figure A.9. In this case the responses of IP are reported instead of employment.

Figure A.11: Impulse response functions from VAR ordering uncertainty first and rv second, using the CBOE VIX



Note: See figure 2. Unlike in the baseline identification, the identified uncertainty shock is not orthogonalized with respect to rv. The rv shock in this case is the remaining part of reduced-form innovation to rv that is not spanned by the uncertainty shock. In this figure, we use the CBOE VIX instead of v_1. Sample period is 1990-2014.

Figure A.12: Impulse response functions from FAVAR



Note: Responses of rv and the first principal component from Ludvigson and Ng (2007) to shocks to rv and the identified uncertainty shock, in a VAR with rv, v_1, federal funds rate, and the PC. The IRFs are scaled so that the two shocks have equal cumulative effects on rv over months 2–24 following the shock. The sample period is 1983–2014. The dotted lines are 68% and 90% confidence intervals.

Table A.1: Predictability of rv with and without v_1 as predictor

Predictors	(1)	(2)	(3)	(4)
rv		0.14		0.09
		(0.09)		(0.09)
v_1		0.56***		0.60***
		(0.14)		(0.15)
FFR	0.01	0.01		
	(0.03)	(0.02)		
Δemp	-31.03*	-2.93		
	(17.67)	(10.92)		
Δip	-5.27	-5.74		
	(5.37)	(3.52)		
PC 1			-0.029**	-0.010
			(0.013)	(0.009)
PC 2			-0.014	-0.017
			(0.018)	(0.012)
PC 3			-0.014	-0.011
			(0.010)	(0.007)
$R_{S\&P}$			-1.56***	0.12
			(0.43)	(0.41)
Def			0.21	0.07
			(0.14)	(0.09)
N	377	377	377	377
Adj. R^2	0.10	0.45	0.20	0.46

Note: Regressions of 6-month realized volatility on lagged rv, option-implied volatility, and various macroeconomic variables, with Hansen-Hodrick standard errors using a 6-month lag window. PC 1 – PC3 are the first three principal components from a large set of macroeconomic time series. $R_{S\&P}$ is the return on the S&P 500. Def is the default spread, the gap between yields on Aaa and Baa bonds.

Table A.2: Predictability of 6-month-ahead v_1

Predictors	(1)	(2)	(3)	(5)	(6)
rv_t	0.02	0.01	0.03	0.01	0.01
	(0.09)	(0.09)	(0.09)	(0.09)	(0.08)
$v_{1,t}$	0.57***	0.49***	0.51**	0.54***	0.60***
	(0.13)	(0.15)	(0.26)	(0.13)	(0.13)
$v_{6,t}$			0.06		
			(0.27)		
rv_{t-1}		0.10*			
		(0.06)			
FFR				0.02	
				(0.015)	
$\Delta empl$				-11.94	
				(9.75)	
Δip				-0.15	
				(3.22)	
PC1					-0.010
					(0.008)
PC2					-0.010
					(0.010)
PC3					-0.008
					(0.006)
$\mathrm{R}_{S\&P}$					0.78**
					(0.38)
Default spread					-0.05
					(0.08)
Adj. R^2	0.35	0.36	0.35	0.37	0.37

Note: Regressions of 6-month-ahead v_1 on lagged rv, option-implied volatility, and various macroeconomic variables, with Hansen–Hodrick (1980) standard errors using a 6-month lag window. PC1–3 are principal components from the data set used in Ludvigson and Ng (2007). The default spread is the difference in yields on Baa and Aaa bonds. The sample is 1983–2014.