How risky is consumption in the long-run? Benchmark estimates from a novel unbiased and efficient estimator

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A large literature studies how risky the economy is. For Epstein–Zin preferences, correct measure is long-run standard deviation (LRSD) of consumption growth. Estimating the LRSD is difficult. This paper: 

- Develops novel non-parametric estimator
- Estimates LRSD with data back to 1834
Epstein–Zin preferences:

\[ V_t = \left\{ (1 - \beta) C_t^{1-\rho} + \beta E_t \left[ V_{t+1}^{1-\alpha} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}} \]

\( \rho \): inverse EIS

\( \alpha \): risk aversion
Price of risk (through the HJ bound) depends on volatility of the SDF
Assume log-normal, homoskedastic consumption growth
Standard deviation of the SDF:

\[
\text{std} \left( M_{t+1} \right) \approx \text{std} \left( \rho \Delta E_{t+1} \Delta c_{t+1} + (\alpha - \rho) \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j \Delta c_{t+1+j} \right)
\]

(exact with unit EIS)

\( \Delta c_t \): log consumption growth
Pricing kernel

Let $\beta \to 1$

\[
\text{std} \left( M_{t+1} \right) \approx \text{std} \left( \rho \Delta E_{t+1} \Delta c_{t+1} + (\alpha - \rho) \Delta E_{t+1} \sum_{j=0}^{\infty} \Delta c_{t+1+j} \right)
\]

- News about $\sum_{j=0}^{\infty} \Delta c_{t+1+j}$ is news about $c_{t+\infty}$
- Most calibrations: $\alpha \gg \rho$

\[
\text{std} \left( M_{t+1} \right) \approx \alpha \times LRSD
\]

- Implies the long-run component dominates
  - Long-run risk model is about making $\Delta E_{t+1} \sum_{j=0}^{\infty} \theta^j \Delta c_{t+1+j}$ very volatile
- $LRSD$ is key to calibrating any model with Epstein–Zin preferences
### Table 1. Recent calibrations of the long-run standard deviation of consumption growth (annualized)

<table>
<thead>
<tr>
<th>Author and Year</th>
<th>Long-run SD</th>
<th>Moments matched</th>
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LRSD appears frequently in econometrics:

- LRSD is the std. dev. of innovations to the Beveridge–Nelson trend (martingale component of $c_t$)
- LRSD determines standard errors in OLS and GMM (e.g. Newey–West estimator)
- Square root of spectral density at frequency zero

Large literature on estimating LRSD
Smoothed periodogram

- Spectral density is $f(\omega)$

\[
LRSD = \sqrt{f(0)}
\]

Need to estimate $f(0)$

- Periodogram is the sample spectrum
  - Defined only at $T-1$ frequencies
  - Measured with error

- Smoothed periodogram estimator:

\[
\hat{f}(0) = \sum_{k=0}^{T-1} K(\omega_k) p(\omega_k)
\]
Benchmark model has strongly peaked spectrum.
Bias and variance

\[ \text{bias} \approx \frac{1}{2} f''(0) \int_{-\pi}^{\pi} \omega^2 K(\omega) \, d\omega \]

\[ \text{variance} \approx \frac{4\pi}{T} f(0)^2 \int_{-\pi}^{\pi} K(\omega)^2 \, d\omega \]

- More peaked kernel:
  - Reduces bias
  - Increases variance

- Changing NW lag length moves along bias/variance tradeoff

- Can we expand the frontier? Yes.
Bias and variance

\[
\text{bias} \approx \frac{1}{2} f'''(0) \int_{-\pi}^{\pi} \omega^2 K(\omega) \, d\omega
\]

\[
\text{variance} \approx \frac{4\pi}{T} f(0)^2 \int_{-\pi}^{\pi} K(\omega)^2 \, d\omega
\]

- If \( K(\omega) \) can be negative, can set approx. bias to zero
- This paper:
  - Set bias to zero
  - Minimize variance
  - Similar to Epanechnikov kernel
- Call it the "high-order kernel"
  - Can then extrapolate to low frequencies
  - Yields lower bias given variance
- High-order estimator yields:
  - Almost exact CI coverage
  - Superior bias/variance tradeoff
Now apply high-order estimator to the data

Three samples:
- Post-war quarterly
- Post-1929 annual
- Post-1834 annual (Barro and Ursua)
Long-run standard deviation estimates and confidence intervals

Quarterly data, 1947–2013
- KL-2010 (8.22)
- BY-2004 (6.28)
- BKY-2010 (4.54)

Annual data, 1929–2012
- 6.61 (95%)
- 5.60 (90%)

Annual data, 1834–2012
- 5.83 (95%)
- 5.36 (90%)

Methods:
- ARMA(4,4)
- ARMA(1,1)
- High-order kernel
- Full-sample point estimate: 4.14% per year
- Post-war data much less volatile
- Conservative LRR calibrations look reasonable
- Parametric estimators yield *much* tighter CI
**Table 1. Recent calibrations of the long-run standard deviation of consumption growth (annualized)**

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Figure 4. Rolling LRSD estimates, 60-year window
Conclusion

- Long-run standard deviation is key moment for models with Epstein–Zin preferences
- Develop novel estimator: lower variance, better confidence interval coverage
- Delivers benchmark estimates of LRSD