Cross-sectional uncertainty and the business cycle: evidence from 40 years of options data

Ian Dew-Becker and Stefano Giglio*

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Abstract

This paper presents a novel and unique measure of cross-sectional uncertainty constructed from stock options on individual firms. Cross-sectional uncertainty varied little between 1980 and 1995, and subsequently had three distinct peaks – during the tech boom, the financial crisis, and the coronavirus epidemic. Cross-sectional uncertainty has had a mixed relationship with overall economic activity, and aggregate uncertainty is much more powerful for forecasting aggregate growth. The data and moments can be used to calibrate and test structural models of the effects of uncertainty shocks. In international data, we find similar dynamics and a strong common factor in cross-sectional uncertainty.

1 Introduction

This paper reports a novel option-implied measure of cross-sectional uncertainty. Whereas the VIX, the most widely used option-implied uncertainty index, measures uncertainty about the state of the aggregate stock market (and, potentially, economy), we construct an index that tracks uncertainty about the cross-sectional distribution of firm outcomes. In many recent models and empirical analyses, it is precisely the cross-sectional component that is the critical driving force.1

*Dew-Becker: Northwestern University and the NBER. Giglio: Yale University. This paper would not have been possible without the version of the Berkeley Options Database preserved and shared by Stewart Mayhew. We thank Terry Hendershott and the Berkeley library for acquiring the data, converting it to a modern format, and making it available.

More formally, one can typically decompose the shock to a firm, \( \eta_{i,t} \), into an aggregate component, \( \mu_t \), and an orthogonal component, \( \varepsilon_{i,t} \) (which may be correlated across subsets of firms)

\[
\eta_{i,t} = \mu_t + \varepsilon_{i,t}
\]

The total uncertainty a firm faces is measured by the conditional (time-\( t \)) variance of \( \eta_{i,t} \). The VIX and other measures of aggregate uncertainty capture the conditional variance of \( \mu_{t+1} \). Finally, cross-sectional uncertainty, on which this paper focuses, is measured by the conditional variance of \( \varepsilon_{i,t} \): it is the variance of the shocks faced by firms that are orthogonal to aggregate shocks. We measure cross-sectional uncertainty similarly to the VIX, using option-implied volatilities.

Our firm-specific implied volatility measure is simple to construct: it is just average firm-level option-implied conditional variance minus market implied conditional variance \( \text{var}_t (\eta_{i,t+1}) - \text{var}_t (\mu_{t+1}) \). Under general conditions, that gap measures the average variance of the residual from a regression of each stock’s return on that of the market. Because it is constructed from market prices, our measure is forward-looking, and is available continuously, in real time, making it particularly useful for policymakers. In addition, the measure is available for a long span of time (40 years), including six recessions. Past work has at most extended to 1996, observing only two business cycles.

In this paper, we document several empirical patterns on the relationship between our new measure of cross-sectional uncertainty and the economy. We focus on two types of patterns: the cyclical behavior of cross-sectional uncertainty and the forecasting power of cross-sectional uncertainty for future economic activity. We find that cross-sectional uncertainty has a mixed relationship with the state of the business cycle, rising during the tech boom of the late 1990’s, but also during the financial crisis and coronavirus epidemic. Figure 1, panel (a), plots cross-sectional uncertainty with a dark line. From the start of our data, in 1980, up to 1995, there was surprisingly little variation. After 1995, firm-level uncertainty moves much more (though still less than market uncertainty, in proportional terms), with three distinct increases, during the tech boom, the financial crisis, and the coronavirus epidemic. In the three episodes where uncertainty is elevated, it rapidly declines, returning to its long-run average by the trough of the recession. In a shorter sample, international data displays similar behavior and also has a very strong factor structure, implying that cross-sectional uncertainty is driven by global shocks.

Overall, the data appears to show that cross-sectional uncertainty is sometimes high in bad times, and sometimes high in good times. Two different classes of models exist that predict one or the other behavior for cross-sectional uncertainty (but not both). The financial
crisis, with low activity and high uncertainty, is consistent with the models that emphasize countercyclical uncertainty, whether it is an endogenous response or an exogenous shock. Interestingly, though, if output tracked cross-sectional uncertainty over time, it would have recovered from the financial crisis by 2010 (when unemployment was still over 9 percent). In contrast to the financial crisis, the period of the late 1990’s is consistent with models in which growth and innovation are associated with uncertainty, e.g. due to learning, creative destruction, or a risk/return trade-off.²

Next, we examine the forecasting power of idiosyncratic uncertainty for aggregate output and employment, finding similarly mixed results. A key feature of the data is that it allows us to test whether aggregate or cross-sectional uncertainty is more relevant for forecasting, which represents a fruitful way to distinguish among classes of structural models and is also relevant for policymakers. We find strong evidence that it is aggregate rather than cross-sectional uncertainty that is most likely to be an important driver of the aggregate economy (though see Berger, Dew-Becker, and Giglio (2020) for questions about the effects of aggregate uncertainty).

We formally examine the cyclicality and forecasting regressions in two theoretical models of the macroeconomic effects of cross-sectional uncertainty shocks: Christiano, Motto, and Rostagno (2014) and Bloom et al. (2018). Both models predict that cross-sectional uncertainty should be clearly countercyclical and should be more tightly related to aggregate output than aggregate uncertainty or realized volatility, inconsistent with the data.

In addition to evaluating correlations and forecasts, the data is also useful for giving a set of moments to aid in calibrating structural models. The data series, available on our websites, gives a direct measure of the underlying driving uncertainty process that needs to be parameterized.

A large literature has studied the relationship between uncertainty and the real economy. However this literature has either focused on aggregate uncertainty, or, if is has looked at individual firms, it have not used forward-looking measures of uncertainty (like ours), but backward-looking measures (realized volatility) that do not map into what uncertainty is in our models.³ The paper shows that distinction changes the conclusions one draws from the data. This is the first work to deliver a long time-series of forward-looking, cross-sectional uncertainty. Only a few papers have similar forward-looking measures of firm-level uncertainty, primarily surveys, but in those cases it is difficult to disentangle the cross-sectional

²See, for example, Acemoglu (2005), Imbs (2007), Comin and Mulani (2009), and Kogan et al. (2017).
³Specifically, Campbell et al. (2001), Bloom (2009), Herskovic et al. (2017), and Bloom et al. (2018) all examine measures of realized dispersion rather than conditional variances. Senga (2018) studies both realized volatility and total firm implied volatility (mixing aggregate and idiosyncratic components) since 1996.
and aggregate components, whereas in the case of stock returns it is straightforward.\footnote{See Guiso and Parigi (1999), Ben-David et al. (2013), Bachmann, Elstner, and Sims (2013), and Bachmann et al. (2018).} This paper’s novelty is in developing an ex ante measure of uncertainty that more directly maps into the shock processes driving structural models.

## 2 Data


The appendix describes the details of the construction of the implied volatilities. Whereas the VIX is measured using a so-called model-free implied volatility, here we just use at-the-money Black–Scholes implied volatility. The latter requires only observing a single option price and is over 99.5 percent correlated with the VIX when they overlap.\footnote{The model-free implied volatility requires a large number of strikes, which is typically not available for individual stocks.} Since implied volatilities come from asset prices, they embed risk premia, regardless of the construction method, meaning they are not errorless measures of investor beliefs. Nevertheless, they represent the single most common real-time measure of uncertainty studied in the literature.

Denote firm $i$’s implied volatility in month $t$ as $\sigma_{i,t}$ and implied volatility for the aggregate stock market as $\sigma_{mkt,t}$. One can always theoretically construct the linear projection of the return on stock $i$, $r_{i,t}$, on the market, $r_{mkt,t}$, as

$$r_{i,t} = \alpha_{i,t} + \beta_{i,t}r_{mkt,t} + \varepsilon_{i,t}$$

with $\varepsilon_{i,t} \perp r_{mkt,t}$ by construction. (2) is just a theoretical representation – it is not directly estimable since the parameters can change on every date, nor is it structural. We follow Campbell et al. (2001) in defining cross-sectional uncertainty as

$$\sigma_{\varepsilon,t}^2 = \sum_i w_i \sigma_{i,t}^2 - \sigma_{mkt,t}^2$$

which is accurate when $\beta_{i,t} \approx 1$. The appendix and figure 2, discussed below, show that the approximation error is in general quantitatively small.

Since $\varepsilon_{i,t}$ is only orthogonal to the market return, it can in general be correlated across firms, e.g. due to industry effects. Changes in the volatilities of cross-sectional factors will appear in $\sigma_{\varepsilon,t}^2$ so we refer to $\sigma_{\varepsilon,t}^2$ as cross-sectional uncertainty.
We measure $\sigma_{mkt,t}^2$ as the at-the-money implied volatility for options on S&P 500 futures. Throughout the analysis, we measure implied volatility interpolated to a maturity of thirty days.

Figure A.1 plots the fraction of total CRSP market capitalization for which we have implied volatilities in each month. For the period covered by the BODB, we have about one third, due to both the fact that not all firms had traded options and that only about half were listed on the CBOE. In 1996, when Optionmetrics becomes available, coverage jumps to 63 percent and then rises to 98 percent by the end of the sample.

Since the BODB has relatively less coverage than Optionmetrics, our main results calculate cross-sectional uncertainty only for the 200 largest firms in the economy over the full sample. Discussed further below, panel (e) of figure 2 shows that the only effect of this choice is to slightly shift the level of implied volatility in the second half of the sample. Since we weight firms by market capitalization, and in any case only have data on publicly traded firms, our results necessarily apply to the largest firms in the economy. These firms account for a large fraction of total economic activity, though, and to the extent that idiosyncratic shocks affect the state of the economy, many theories imply it will be the largest firms whose shocks pass through to the aggregate economy (e.g. Gabaix (2011) and Acemoglu et al. (2012)).

3 Time-series behavior of cross-sectional uncertainty

3.1 Univariate behavior and cyclicality

3.1.1 Variability

Panel (a) of figure 1 plots the time series of cross-sectional uncertainty, $\sigma_{\epsilon,t}$. In the first half of the sample, there is remarkably little variation: its standard deviation is only 10 percent of its mean for the period 1980–1997. But since 1997, rose by nearly a factor of four to 37 percent of its mean.

$$SD(\sigma)/E(\sigma):$$

For BODB, tickers must be matched by hand to CRSP to obtain underlying stock prices. We did that only for the top 200 firms by size, which account for the vast majority of the market capitalization in the sample.
<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{\epsilon,t}$</th>
<th>$\sigma_{mkt,t}$</th>
</tr>
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<tbody>
<tr>
<td>Full sample:</td>
<td>0.40</td>
<td>0.29</td>
</tr>
<tr>
<td>1980–1997</td>
<td>0.30</td>
<td>0.09</td>
</tr>
<tr>
<td>1998–2020</td>
<td>0.43</td>
<td>0.37</td>
</tr>
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</table>

Panel (a) of figure 1 also reports the implied volatility for the overall stock market. Relative to its mean, aggregate uncertainty is substantially more variable than cross-sectional uncertainty, which is also apparent from the table above. The standard deviation of $\sigma_{mkt}$ is 40 percent of its mean overall, compared to only 29 percent for cross-sectional volatility. In absolute terms, though, the standard deviations of the two series are nearly identical over the full sample.

The variability of $\sigma_{mkt,t}$ is also much less isolated in time than that of $\sigma_{\epsilon,t}$. Whereas the variation in cross-sectional uncertainty is driven primarily by just three episodes, there are numerous substantial jumps in market-level uncertainty, associated with the 1987 stock market crash, the first Gulf War, various events between 1998 and 2002, the 2008 financial crisis, the debt ceiling, the Euro crisis, and coronavirus.

The relative volatilities of $\sigma_{\epsilon,t}^2$ and $\sigma_{mkt,t}^2$ can be used to construct a variance decomposition for the total variance faced by firms. Specifically,

$$\text{var} \left( \sum_i w_{i,t} \sigma_{i,t}^2 \right) = \text{var} \left( \sigma_{mkt,t}^2 \right) + \text{var} \left( \sigma_{\epsilon,t}^2 \right) + 2 \text{cov} \left( \sigma_{i,t}^2, \sigma_{mkt,t}^2 \right) = 1.43 \times 10^{-2} = 0.45 \times 10^{-2} + 0.46 \times 10^{-2} + 0.52 \times 10^{-2}$$

Over the sample, the variation in the total uncertainty that firms face is essentially equally driven by variation in aggregate and cross-sectional uncertainty. This again masks differences between the first and second halves of the sample – in the first half firm uncertainty almost entirely depended on aggregate shocks, while in the second half cross-sectional uncertainty became more important.

The moments reported here on the volatilities of aggregate and cross-sectional uncertainty are useful for calibrating structural models of uncertainty shocks. We return to this point below.

### 3.1.2 Cyclicality

Figure 1 (panel (b)) adds further context to the time series of uncertainty by plotting cross-sectional uncertainty against the detrended level of the CRSP total stock market index. The periods of high cross-sectional uncertainty are all associated with large changes in stock prices, but in opposite directions. During the dot-com boom, cross-sectional tracks the rise of the stock market. They peak in almost exactly the same month, and cross-sectional
volatility declines in unison with the level of the market. In other words, uncertainty is high in the late 1990’s when stock prices are rising, and it falls as conditions deteriorate. It follows the opposite pattern during the financial crisis: it is exactly when the stock market begins to decline that cross-sectional uncertainty rises. So uncertainty appears to be procyclical in the late 1990s and early 2000s, countercyclical in the financial crisis and in the recent coronavirus episode, and acyclical otherwise.

Panel (d) of 1 further emphasizes that point by plotting cross-sectional uncertainty against aggregate investment. Investment and uncertainty peak simultaneously in 2000, while uncertainty spikes and investment crashes in 2008. Panel (a) of figure 2 shows that uncertainty has a similarly mixed relationship with the unemployment rate.

To more formally quantify the cyclicality of cross-sectional uncertainty, panel (a) of table 1 reports the correlation of cross-sectional uncertainty with various measures of the state of the economy, over both the full sample and also pre- and post-1998. In terms of levels, cross-sectional uncertainty does not have a consistent correlation with economic indicators. It is positively correlated with the CBO output gap, detrended employment and minus the unemployment rate, implying it is procyclical. Its correlations with detrended industrial production and capacity utilization are close to zero, and its correlation with the Gilchrist–Zakrajsek bond spread is positive, implying it is countercyclical. So relative to levels, it appears essentially acyclical. Furthermore, each of those correlations reverses sign between the first and second halves of the sample, emphasizing their overall indeterminacy.

Relative to growth rates, uncertainty appears more consistently countercyclical, with negative correlations with IP growth, employment growth, the change in the output gap, minus the unemployment rate, and a recession indicator. Those correlations also generally retain the same sign in the first and second halves of the sample.

Finally, it is noteworthy how quickly cross-sectional uncertainty declines following its peaks in 2000 and 2008. In both cases, cross-sectional uncertainty falls back to its long-run average much more quickly than market-level uncertainty does.

Overall, then, to the extent that there is a clear cyclicality for cross-sectional uncertainty, it is that high uncertainty is associated with turning points. In levels, it has a weak relationship with the state of the economy. But relative to changes the correlations are much stronger. That is consistent with the fact that its increases are short-lived – when it has risen in recessions, it did so only at the beginning and quickly fell back towards its mean.

### 3.2 Robustness and further results

In panels (b)-(f), figure 2 plots five variations on the benchmark uncertainty series:
1. Using the median of implied volatility across firms (after taking out firm fixed effects), instead of weighting by market capitalization (though note this still only applies to the top 200 firms by market capitalization).

2. Correcting for each firm’s loading on the market, by estimating $\beta_i$ for each firm and setting $\sigma^2_{\varepsilon,i,t} = \sigma^2_{i,t} - \beta_i^2 \sigma^2_{mkt,t}$.

3. Reweighting the sample so that the total weight of each three-digit NAICS sector gets the same weight in the calculation of $\sigma^2_{\varepsilon,t}$ as its employment weight in the corresponding month.

4. Using the full sample of options from Optionmetrics instead of just the largest 200 firms.

5. Controlling for industry effects by setting $\sigma^2_{\varepsilon,i,t} = \sigma^2_{i,t} - \sigma^2_{ind(i),t}$, where $\sigma^2_{ind(i),t}$ is the implied variance for the SPDR exchange traded fund covering stock $i$’s sector.

The first and second tests show that the results are not driven just by the weighting by market capitalization or the use of the very largest stocks. When using the full Optionmetrics sample, the cross-sectional implied volatility shifts up slightly, but its time-series variation is essentially unchanged – the benchmark series is over 99 percent correlated with the calculation from the full sample.

The third test shows that the approximation where we treat the loadings on the market as all equal to 1 has very little impact. The fourth test shows that even though the market cap weights are not the same as employment weights across sectors, the effect on the overall cross-sectional volatility is not particularly noteworthy. The largest effect is during the tech bubble, when tech stocks had high market capitalization relative to their share of the real economy and also relatively high implied volatilities. Fifth, removing the industry component of volatility also does not qualitatively change the conclusions. Its main effect is to dampen the cross-sectional uncertainty in the tech boom and financial crisis, consistent with those episodes being at least partially by contributions from specific industries – tech and finance.

Appendix A.2 discusses the commonality in variation in uncertainty across firms. Consistent with Herskovic et al. (2017), 40–50 percent of cross-sectional variation in firm-specific uncertainty is captures by the common component.

Finally, in results available on request, we have also constructed firm-specific uncertainty using the so-called model-free implied volatility (used in the construction of the S&P 500 VIX) instead of at-the-money implied volatility and obtain nearly identical results.
3.3 Uncertainty versus realized dispersion

Prior to this paper, the available data on forward-looking uncertainty covered at most only two recessions, making it relatively uninformative about the cyclical nature of uncertainty. That is, past studies have used realized dispersion as a proxy for $\sigma^2_{\hat{\epsilon},t}$. That is, the existing literature has used the cross-sectional standard deviation of the realizations of $\hat{\epsilon}_{i,t}$ instead of $\sigma^2_{\hat{\epsilon},t}$ to capture the time variation in cross-sectional uncertainty. The difference between the two is not innocuous.

Panel (c) of figure 1 plots $\sigma^2_{\hat{\epsilon},t}$ against the realized cross-sectional standard deviation of the firm-specific residuals, $\hat{\epsilon}_{i,t}$ (the latter has been analyzed, for stock returns, by Campbell et al. (2001) and Herskovic et al. (2017), among others). The realized standard deviation behaves substantially differently from $\sigma^2_{\hat{\epsilon},t}$. In particular, it appears to have a substantial amount of high-frequency noise. That is consistent with the fact that $\sigma^2_{\hat{\epsilon},t}$ is equal to the expectation of realized dispersion. There is also a clear difference in the means of the two series, which is explained by a risk premium on realized dispersion. Importantly, there does not appear to be substantial variation in that premium.

Note that the two series are related to each other: $\sigma^2_{\hat{\epsilon},t-1}$ is the conditional expectation of the realized dispersion in returns, defined as:

$$RD^{ret}_t = \sum_i w_{i,t} \text{var}(r_{i,t}) - \text{var}(r_{mkt,t})$$

where $\text{var}(r_{i,t})$ and $\text{var}(r_{mkt,t})$ are calculated from returns within each month. Therefore, we have:

$$RD^{ret}_t = \sigma^2_{\hat{\epsilon},t-1} + \eta_t$$

where $\eta_t$ is a mean-zero shock that is uncorrelated with $\sigma^2_{\hat{\epsilon},t-1}$.

Equation (5) means that if one’s goal is to understand the behavior of uncertainty (the forward-looking $\sigma^2_{\hat{\epsilon},t-1}$) – its variability, correlation with other variables, or its coefficient in forecasting regressions (below) – then proxying for it with $RD^{ret}_t$ will cause biases. The volatility of $RD^{ret}_t$ is substantially higher than that of $\sigma^2_{\hat{\epsilon},t-1}$, its correlation with other variables is lower, and in regressions there will be an attenuation bias even if $\eta_t$ is exogenous (due to a classical errors-in-variables problem). If $\eta_t$ is correlated with outcomes of interest, that will further bias any regressions. Berger, Dew-Becker, and Giglio (2020), for example, show that when structural productivity shocks are skewed left (consistent with observed asymmetry in the business cycle), then realized volatility, $\eta_t$, will be negatively correlated with output, even if there is no structural effect of uncertainty on activity.

Furthermore, for a policymaker working in real time, the relative precision of $\sigma^2_{\hat{\epsilon},t}$ is an
important added advantage. If one’s goal is to know what uncertainty is right now, the added noise in realized dispersion – the $\eta_t$ term – makes it less useful than the option-implied true uncertainty $\sigma_{\varepsilon,t}$.

4 The predictive power of uncertainty

4.1 Forecasting realized dispersion

We now examine the forecasting power of uncertainty. The first question is whether uncertainty is actually useful for forecasting future realized dispersion, as predicted by equation (5). More importantly, does option-implied uncertainty contain information not available from the lags of past realized dispersion? If $RD_{t+1}^{ret}$ followed an autoregressive process, for example, it would not. We measure realized dispersion at the quarterly level in these regressions, which makes it consistent across our different variables.

Panel (b) of table 1 reports results of regressions of realized dispersion on the lag of $\sigma_{\varepsilon,t}$. In all cases, both the dependent and independent variables are standardized to have unit standard deviations. The first column shows that $\sigma_{\varepsilon,t}$ has substantial forecasting power for $RD_{t+1}^{ret}$, in fact driving the lagged value of realized dispersion out of the regression. So not only does $\sigma_{\varepsilon,t}$ contain marginal information, but it in fact is dominant.\footnote{The constant in the regression is not equal to zero, consistent with the presence of the risk premium apparent from figure 1, panel (c).}

The second two columns report results for two alternative measures of cross-sectional dispersion: the cross-sectional interquartile ranges of growth in industrial production (across sectors) and growth in sales (across COMPUSTAT firms), $RD_{t+1}^{IP}$ and $RD_{t+1}^{sales}$. In both cases, $\sigma_{\varepsilon,t}$ again has substantial forecasting power, with even the lower end of the confidence bands for the coefficients nontrivially positive. In these cases the coefficients no longer drive out lagged realized dispersion, but that is not surprising given that $\sigma_{\varepsilon,t}$ directly measures stock return uncertainty, not uncertainty for IP or sales.

These regressions show that $\sigma_{\varepsilon,t}$ is in fact a true measure of forward-looking uncertainty in the sense that it forecasts the future cross-sectional dispersion in outcomes, even controlling for lagged dispersion. For stock returns, in particular, it actually drives lagged realized dispersion out of the regression.
4.2 Forecasting real activity

We now examine the ability of cross-sectional uncertainty to forecast three standard measures of real activity available at the monthly frequency: the unemployment rate, non-farm private employment growth, and industrial production growth. All of the variables are again standardized to have unit variance to aid interpretation. These regressions are valuable for two reasons: they represent a moment that can also be calculated in models as a test, and they are useful on their own for the purpose of forecasting, e.g., for policymaking. They are distinct from the results on cyclicality in the previous section because we are now asking about the information content of uncertainty for future rather than contemporaneous outcomes.

The first column in each of the three sections of table 1, panel (c), reports a regression of activity on lagged cross-sectional uncertainty. In all three cases, the coefficient implies that increases in uncertainty are followed by declines in real activity. So, consistent with the previous results, there is a relationship between cross-sectional uncertainty and changes in real activity. The magnitudes of the coefficients are similar, with a unit standard deviation increase in uncertainty being associated with declines in IP and employment of about 0.14 standard deviations and an increase in unemployment of 0.09 standard deviations.

While these univariate results are informative, sharper tests of theoretical models can be obtained by contrasting the forecasting power of cross-sectional uncertainty with that of other measures, like realized cross-sectional dispersion and aggregate uncertainty. Empirically, these measures are potentially correlated, but the existing literature has often offered sharp predictions about which type of volatility matters for the real economy.

Studying the effects of aggregate (not cross-sectional) uncertainty, Berger, Dew-Becker, and Giglio (2019) show that in similar forecasting regressions, uncertainty is driven out by realized volatility, which is the aggregate analog to realized cross-sectional dispersion. Given this result, it is natural to ask whether in our context as well realized dispersion drives out cross-sectional uncertainty. The second column of each section of table 1, panel (c), shows that when realized dispersion is included in the forecasting regressions, it is associated with declines in both employment and industrial production and increases in unemployment, with larger coefficients than cross-sectional uncertainty.

These results, together with the ones in the previous section, tell us that even though forward-looking cross-sectional uncertainty $\sigma_{\varepsilon,t}$ is a better measure of uncertainty than realized dispersion $RD_{t}^{ret}$ (lagged RD is driven out by $\sigma_{\varepsilon,t}$ when predicting future RD), $RD_{t}^{ret}$ has more forecasting power for real activity. This suggests that unanticipated shocks to realized dispersion—what appears in panel (c) of figure 1 to be just noise in $RD_{t}^{ret}$—relative
to $\sigma_{\varepsilon,t}$ – rather than true uncertainty shocks (that is, changes in expected dispersion), is more important for explaining economic fluctuations.

Similarly, table 1 also asks whether cross-sectional or aggregate uncertainty is most important for forecasting. The third column in each section of panel (c) shows that in all three cases, market uncertainty drives cross-sectional uncertainty out of the regressions. Not only is market uncertainty dominant in relative terms, its coefficients are also very large in absolute terms, ranging in magnitude from 0.19 to 0.35, or twice as large as the coefficients on $\sigma_{\varepsilon,t}$ in the first column. The data therefore shows that to the extent that uncertainty is relevant for forecasting real activity, it is aggregate rather than cross-sectional uncertainty that matters. That result has significant implications for structural models of uncertainty shocks.

Overall, then, while cross-sectional uncertainty does have some univariate forecasting power, it is delicate. In addition, when realized dispersion is included, it typically has a larger coefficient, and when aggregate uncertainty is included, it becomes dominant.\(^8\)

5 Calibrating and testing structural models

The analysis so far has analyzed cross-sectional uncertainty purely empirically, examining its univariate behavior and relationship with other variables. This section examines the use of $\sigma_{\varepsilon,t}$ for calibrating and testing structural models. Having a time-series for $\sigma_{\varepsilon,t}$ allows us to measure the time-series dynamics of uncertainty, which is a key input to calibrations of models. Furthermore, regressions like the forecasting exercises in table 1 are a useful test of structural models. We focus on two recent prominent models: the “really uncertain business cycles” (RUBC) model of Bloom et al. (2018), which is centered around a real options framework, and the model of financial frictions of Christiano, Motto, and Rostagno (CMR; 2016). Since both of the models are constructed at the quarterly frequency, the empirical results reported in table 2 are also measured at the quarterly frequency.

5.1 Calibration moments

In the vast majority of models with time-varying idiosyncratic uncertainty, uncertainty follows an AR(1) type process. Such a process can be characterized, up to a scaling term, by its standard deviation (which we scale relative to its mean to account for leverage and other

\(^8\)Interestingly – but outside the scope of this paper – the results in Berger, Dew-Becker, and Giglio (2020) further show that aggregate uncertainty itself is driven out of forecasting regressions by aggregate realized volatility.
scaling factors) and its autocorrelation. Panel (a) of table 2 reports those moments in the data, along with bootstrapped 95-percent confidence bands. These numbers are useful both as a guide for future calibrations and also for evaluating calibrations used in past work. As discussed above, the standard deviation of cross-sectional uncertainty relative to its mean is 29 percent over the full sample. Its quarterly autocorrelation (chosen to match the most common calibration frequency in the literature) is 0.906.

The next six columns of the table report analogous population moments in the calibrations of six recent structural models. In all six cases, the idiosyncratic uncertainty process represents the volatility of firm-specific shocks to fundamentals (typically productivity). For the scaled standard deviation, the calibrations range from 0.093 to 0.71. Of the six publications examined, the scaled standard deviation lies inside the confidence band for our data in only two cases. For the autocorrelation, the results are similar. There is again a very wide range across the calibrations, from 0.707 to 0.983, and in only three of the six cases is the calibrated value inside the confidence bands. The table therefore shows that there is little agreement in the literature on either the volatility or persistence of idiosyncratic risk. The data presented here can help resolve that disagreement.

5.2 Correlations

Panel (b) of table 2 reports raw correlations between major economic aggregates and uncertainty at the quarterly frequency. As in table 1, the correlations are weak and have mixed signs. The second two columns report population correlations in RUBC and CMR. In both cases, with the exception of consumption growth, the correlations are substantially negative and well outside the empirical confidence bands. Both models focus on contractionary effects of uncertainty shocks. While the data presented here is consistent with the existence of such mechanisms during some episodes, in that uncertainty was high during the 2009 and 2020 recessions, the large increase in cross-sectional uncertainty during the boom of the late 1990’s renders the overall correlation close to zero. Matching the two types of behavior simultaneously appears to still be beyond the state-of-the-art models.

5.3 Forecasting regressions

A third way that the data can be used to evaluate models is to ask whether they can match the forecasting results obtained in table 1. Recall that in addition to finding that cross-sectional uncertainty alone had some forecasting power, we also found that it was driven out by both realized cross-sectional dispersion and aggregate uncertainty. The replication files
for CMR include time series for both ex-ante cross-sectional uncertainty, corresponding to \( \sigma_{\epsilon,t} \), and also realized dispersion, which we map to \( RD_{t}^{ret} \). We can therefore estimate, in both the model and the data, regressions of real activity on uncertainty and realized dispersion to measure which is more important for forecasting.

To help particularly focus on the effects of uncertainty shocks, in both the model and data here we regress changes in macro aggregates on changes in uncertainty and realized dispersion. For CMR, then, the regression we estimate in the data and model is

\[
\Delta y_t = b_0 + b_1 \Delta y_{t-1} + b_2 \Delta \sigma_{\epsilon,t-1} + b_3 \Delta RD_{t-1}^{ret} + \eta_t
\]  

(6)

where \( \eta_t \) is a residual and \( y \) represents the log of GDP, consumption, investment, or hours worked. As in the analysis above, all variables are standardized to have unit standard deviation.

The first column in panel (c) of table 2 reports the estimates from the data, while the second column reports the true (population) coefficients in CMR. In all four cases, there is again only a weak relationship in the data between economic activity and shocks to uncertainty – for investment, the relationship is actually positive – while realized dispersion has a statistically significantly negative relationship with real activity. In CMR, on the other hand, in all four cases it is uncertainty that is most important for forecasting real activity, rather than realized dispersion. That fact is not surprising in the model. As discussed above, realized dispersion is equal to uncertainty plus noise (the unexpected component of realized dispersion). When that noise does not have structural effects, as in CMR, then uncertainty will dominate the forecasts. That is exactly what we find in the model.

In the data, though, that “noise” – the gap between realized dispersion and its expectation – actually contains information. Models featuring concave responses to shocks, such as Ilut, Kehrig, and Schneider (2018) and Dew-Becker, Tahbaz-Salehi, and Vedolin (2020), are able to generate that effect. That is, in those models, realized dispersion does have effects on output, above and beyond the expected component encoded in uncertainty.

As discussed above, given that with our data we can construct a pair of matching measures of both aggregate and cross-sectional uncertainty, we can address the question of which of the two is more important for driving fluctuations. The second pair of columns in table 2, panel (c), reports estimates from the regression

\[
\Delta y_t = b_0 + b_1 \Delta y_{t-1} + b_2 \Delta \sigma_{\epsilon,t-1} + b_3 \Delta \sigma_{mkt,t-t} + \eta_t
\]  

(7)

Compared to above, we have replaced \( RD_{t}^{ret} \) with \( \sigma_{mkt,t-t} \). As in table 1, panel (c) of table
2 shows that aggregate uncertainty drives cross-sectional uncertainty out of the forecasting regressions. Again, GDP, investment, and hours are all substantially more strongly driven by shocks to aggregate than cross-sectional uncertainty.

While CMR only has time-varying cross-sectional uncertainty, RUBC has fluctuations in both cross-sectional and aggregate uncertainty. In that paper’s benchmark calibration, cross-sectional and aggregate uncertainty are perfectly correlated. We run the simulation code three different times with calibrations where cross-sectional and aggregate uncertainty vary by different amounts, so that when the three calibrations are combined, the perfect correlation is broken.\textsuperscript{9}

In panel (c) of table 2, the coefficients on aggregate uncertainty in the forecasting regression run on simulations of the RUBC model are, instead of being dominant, in all cases much smaller than the coefficients on cross-sectional uncertainty, by factors of three to four. In the model, it is primarily variation in cross-sectional uncertainty that matters. That fact makes sense given that in the model, the vast majority of the variation in the total uncertainty faced by firms is from the cross-sectional component.

5.4 Implications

To sum up, table 2 makes two basic contributions. The top panel is useful for giving specific moments for calibrating structural models. For example, if uncertainty follows an AR(1) process, the volatility and persistence can be chosen to match the moments for the data presented here.

The second contribution is to show how the raw correlations and forecasting regressions provide insights into aspects of the data that models can and cannot match. The data implies that realized dispersion is at least as important for forecasting as ex-ante uncertainty, if not more important. Similarly, it strongly implies that aggregate uncertainty is more important than cross-sectional uncertainty. Both of those features of the data are difficult for two leading models to match, so future work could use them as areas for improvement. The forecasting regressions suggest that it is aggregate uncertainty – associated with unhedgeable shocks – rather than cross-sectional uncertainty, which can be hedged, that agents are responding to, if anything.

As discussed above, the targets in panels (b) and (c) of table 2 are difficult to match within a single model – they require being able to match the fact that uncertainty is sometimes

\textsuperscript{9}In the baseline RUBC calibration, aggregate and cross-sectional uncertainty rise by a factors of 1.61 and 4.14, respectively, in the high-uncertainty state. We construct two additional simulations in which they rise by the factors \{2.42, 2.76\} and \{1.07, 6.21\} and append them to the baseline.
good and sometimes bad, and also being able to explain why realized dispersion would have independent effects (which may require a nonlinear model). It is no criticism of RUBC and CMR that their benchmark calibrations do not match these new results. Rather, table 2 simply yields new empirical facts that structural models can be built or enriched to match.

6 International evidence

As in the US analysis, we obtain data from Optionmetrics Europe for 1/2002 to 12/2018 and from Bloomberg for 1/2019 to 5/2020. Data on market capitalization for individual firms is obtained from the Compustat Global database. We have acceptable data for Switzerland, Germany, Spain, France, Great Britain, and the Netherlands. While all of the countries are from western Europe, the list includes countries with varying degrees of connection to the EU and countries on different currencies and with very different government fiscal states.

Figure 3 plots the time series of cross-sectional uncertainty for each country against that for the US. Across all six countries, cross-sectional uncertainty is clearly strongly correlated with that in the US. In all six countries, cross-sectional uncertainty is elevated in 2002, declines until rising during the financial crisis, and then stays low and stable from 2010 to the appearance of coronavirus in 2020. That is true even though the path of aggregate output in Europe over this period was very different from the US – a number of these countries went into recessions around 2012.

To formalize those results, each panel reports the correlation of the country’s cross-sectional uncertainty with that of the US. Across the six countries, the average correlation is 0.80 (for market uncertainty the average correlation is 0.91). Those numbers imply that US uncertainty explains on average 65 percent of the variation in cross-sectional uncertainty (and any measurement error in the uncertainty series will bias that number down).

The table below summarizes the various results for market and cross-sectional uncertainty. It shows that the average time series standard deviation of market uncertainty is nearly twice that of cross-sectional uncertainty. Furthermore, it confirms the results on the fraction of the variation in uncertainty explained by a common factor – this time using the cross-sectional mean of uncertainty, rather than the US value. Finally, it reports the simple average of all the pairwise correlations across countries and shows the high degree of similarity for both types of uncertainty.

Statistics for cross-sectional and market uncertainty across countries
Cross-sectional unc. | Market unc.
---|---
Avg. time-series s.d. | 0.0474 | 0.0803
Avg. cross-sectional s.d. | 0.0342 | 0.0322
Frac. of var. explained by cross-sectional mean | 0.60 | 0.86
Avg. pairwise corr. | 0.82 | 0.92

The results here are important for two reasons. First, they show that the finding of stability is not unique to the US: cross-sectional uncertainty has been similarly stable in other major developed economies. Second, they show that there appears to be a very strong international factor in cross-sectional uncertainty: uncertainty shocks have been global in nature over the last 18 years.

The latter result is particularly surprising. It is simple to envision a model in which there is a common component in market level uncertainty (e.g. Herskovic et al. (2017)). That will happen whenever there is a heteroskedastic shock that affects stock prices globally. A trade war, for example, might generally be expected to negatively impact all economies, so when there is more uncertainty about trade we would expect market uncertainty to rise everywhere.

Common variation in that cross-sectional uncertainty, on the other hand, must come from common movements across countries in the volatility of shocks that affect relative performance across firms. Whatever the shocks are that cause these cross-sectional movements – shifts in creative destruction, for example – they must occur across the various economies that we study here simultaneously.

7 Conclusion

This paper reports a novel real-time index of cross-sectional implied volatility. A large literature studies the effects – both good and bad – of variation in the cross-sectional distribution of shocks that firms face. There is theoretical ambiguity about the effects of changes in cross-sectional uncertainty, but many policymakers take the view that uncertainty represents a hindrance to economic growth. It is thus an important empirical question not just what the time series of firm-level of uncertainty has looked like, but also whether shocks to cross-sectional uncertainty are in fact contractionary.

We develop a novel index of cross-sectional uncertainty with data extending back to 1980. The length of the sample is important – it is the data in the 1980’s and early 1990’s that emphasizes the extent to which the last two recessions have been anomalous. Prior to
the late 1990’s, there was little variation in cross-sectional uncertainty. Since then, there have been three episodes where it substantially grew, one a major economic expansion and the other two contractions. Studying raw correlations and forecasting regressions, we find that cross-sectional uncertainty is approximately acyclical and has little ability to forecast changes in future real activity. Overall, sometimes the data is consistent with models in which uncertainty shocks have causal negative effects on the economy, while in other periods it is consistent with models in which cross-sectional uncertainty is high following good shocks, perhaps due to a rise in creative destruction.

References


Figure 1: Time series of cross-sectional uncertainty

(a) Cross-sectional unc. and aggregate uncertainty
(b) Cross-sectional unc. and CRSP market value
(c) Cross-sectional unc. and realized dispersion
(d) Cross-sectional unc. and investment

Note: Each panel plots cross-sectional uncertainty (darker line) together with another time series: aggregate uncertainty (panel (a)), stock market value (panel (b)), realized dispersion (panel (c)), and investment (panel (d)). Options data before 1996 is from the Berkeley Options Dataset, for the period 1996/01-2019/06 is from Optionmetrics, and after 2019/06 is from Bloomberg. The VIX (market uncertainty) is obtained from CME options. Shaded areas are NBER recessions.
Figure 2: Robustness

(a) Cross-sectional unc. and the unemployment rate
(b) Baseline vs. median IV
(c) Baseline vs. beta-adjusted
(d) Baseline vs. employment weights
(e) Baseline vs. all Optionmetrics
(f) Baseline vs. industry-adjusted

Note: Panel (a) plots cross-sectional uncertainty together with the unemployment rate. Panel (b)-(f) plot our baseline measure of cross-sectional uncertainty together with alternative measures built in different ways. Specifically: in panel (b), the alternative measure uses the median of implied volatility across firms, instead of the weighted average by market cap; in panel (c), the alternative adjusts for betas; in panel (d), it uses employment weights; in panel (e), it uses all options in Optionmetrics instead of the largest 200; in panel (f), it controls for industry effects.
Figure 3: Cross-sectional uncertainty across countries

**Note:** Cross-sectional uncertainty from option data in European markets (solid line) against the one for the US (dotted line). Data is from Optionmetrics until 2018, and from Bloomberg since 2019.
Table 1: Cyclicality of cross-sectional regressions and forecasting results

Panel (a): Correlations

<table>
<thead>
<tr>
<th></th>
<th>Detrended IP</th>
<th>Detrended empl.</th>
<th>Unemployment rate</th>
<th>CBO output gap</th>
<th>NBER recession</th>
<th>Capacity utilization</th>
<th>CZ bond spread</th>
<th>IP growth</th>
<th>Employment growth</th>
<th>Change in unempl. rate</th>
<th>Change in output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>0.01</td>
<td>0.22</td>
<td>-0.22</td>
<td>0.36</td>
<td>0.03</td>
<td>0.48</td>
<td>-0.20</td>
<td>-0.30</td>
<td>0.26</td>
<td>-0.22</td>
<td></td>
</tr>
<tr>
<td>Pre-1998</td>
<td>-0.34</td>
<td>-0.44</td>
<td>0.42</td>
<td>0.28</td>
<td>-0.24</td>
<td>-0.16</td>
<td>-0.02</td>
<td>-0.15</td>
<td>0.06</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Post-1998</td>
<td>0.03</td>
<td>0.31</td>
<td>-0.26</td>
<td>0.29</td>
<td>0.39</td>
<td>0.11</td>
<td>0.53</td>
<td>-0.23</td>
<td>-0.34</td>
<td>0.31</td>
<td>-0.28</td>
</tr>
</tbody>
</table>

Panel (b): Forecasting realized dispersion

\[ RD_t^x = b_0 + b_1 RD_{t-1}^x + b_2 \sigma_{\epsilon,t-1} + \eta_t \]

<table>
<thead>
<tr>
<th></th>
<th>Stock return realized dispersion</th>
<th>IP growth IQR</th>
<th>Sales growth IQR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\epsilon,t-1} )</td>
<td>0.69</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td>#obs.</td>
<td>143</td>
<td>142</td>
<td>141</td>
</tr>
</tbody>
</table>

Panel (c): Uncertainty forecasting real activity

\[ y_t = b_0 + b_1 y_{t-1} + b_2 \sigma_{\epsilon,t-1} + b_3 RD_{t-1}^{ret} + b_4 \sigma_{mkt,t-1} + \eta_t \]

<table>
<thead>
<tr>
<th></th>
<th>unemp</th>
<th>Demp</th>
<th>Dip</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\epsilon,t-1} )</td>
<td>0.09</td>
<td>-0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td>#obs.</td>
<td>442</td>
<td>434</td>
<td>442</td>
</tr>
<tr>
<td>( RD_{t-1}^{ret} )</td>
<td>0.04</td>
<td>-0.02</td>
<td>-0.04</td>
</tr>
<tr>
<td>#obs.</td>
<td>442</td>
<td>434</td>
<td>442</td>
</tr>
<tr>
<td>( \sigma_{mkt,t-1} )</td>
<td>0.19</td>
<td>-0.35</td>
<td>-0.24</td>
</tr>
<tr>
<td>#obs.</td>
<td>442</td>
<td>434</td>
<td>442</td>
</tr>
</tbody>
</table>

Note: Panel (a) reports correlations between cross-sectional uncertainty and various macroeconomic variables. Panel (b) reports the results of a regression of three different measures of realized cross-sectional dispersion on lagged cross-sectional uncertainty and lagged realized dispersion. The three measures are: realized cross-sectional dispersion of stock returns, the cross-sectional interquartile ranges of growth in industrial production (across sectors), and growth in sales (across COMPUSTAT firms). Panel (c) reports forecasting regressions of real activity, in the three sections, respectively: unemployment, change in employment, and change in industrial production. In each section, the first column uses lagged cross-sectional uncertainty as predictor, the second column adds lagged realized cross-sectional dispersion, and the last column adds instead market-wide uncertainty.
Table 2: Comparing models to data moments

Panel A. Calibration moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RUBC</th>
<th>CMR</th>
<th>Schaal</th>
<th>Di Tella</th>
<th>Gilchrist et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( SD[\sigma_{\varepsilon,t}] / E[\sigma_{\varepsilon,t}] )</td>
<td>0.29</td>
<td>0.71</td>
<td>0.58</td>
<td>0.136</td>
<td>0.203</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.16,0.37]</td>
</tr>
<tr>
<td>( Corr[\sigma_{\varepsilon,t}, \sigma_{\varepsilon,t-1}] )</td>
<td>0.91</td>
<td>0.91</td>
<td>0.98</td>
<td>0.937</td>
<td>0.707</td>
<td>0.902</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.86,0.96]</td>
</tr>
</tbody>
</table>

Panel B. Correlations of growth rates with \( \Delta \sigma_{\varepsilon,t} \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>RUBC</th>
<th>CMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>-0.04</td>
<td>-0.53</td>
<td>-0.39</td>
</tr>
<tr>
<td></td>
<td>[-0.17,0.12]</td>
<td>[-0.08,0.32]</td>
<td>[-0.10,0.10]</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.10</td>
<td>0.37</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>[-0.08,0.32]</td>
<td>[0.12,0.08]</td>
<td>[-0.10,0.10]</td>
</tr>
<tr>
<td>Investment</td>
<td>-0.01</td>
<td>-0.59</td>
<td>-0.43</td>
</tr>
<tr>
<td></td>
<td>[-0.10,0.10]</td>
<td>[-0.09,0.25]</td>
<td>[-0.10,0.10]</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.10</td>
<td>-0.76</td>
<td>-0.21</td>
</tr>
<tr>
<td></td>
<td>[-0.09,0.25]</td>
<td>[-0.09,0.25]</td>
<td>[-0.09,0.25]</td>
</tr>
</tbody>
</table>

Panel C. Forecasting regressions

\[
CMR: \, y_t = b_0 + b_1 y_{t-1} + b_2 \sigma_{\varepsilon,t-1} + b_3 R D_{t-1}^{ret} + \eta_t
\]

\[
RUBC: \, y_t = b_0 + b_1 y_{t-1} + b_2 \sigma_{\varepsilon,t-1} + b_3 \sigma_{mkt,t-1} + \eta_t
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>CMR</th>
<th>Data</th>
<th>RUBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>( \sigma_{\varepsilon,t-1} )</td>
<td>-0.002</td>
<td>-0.24</td>
<td>( \sigma_{\varepsilon,t-1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>( \sigma_{\varepsilon,t-1} )</td>
<td>-0.005</td>
<td>-0.04</td>
<td>( \sigma_{\varepsilon,t-1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hours</td>
<td>( \sigma_{\varepsilon,t-1} )</td>
<td>0.017</td>
<td>-0.23</td>
<td>( \sigma_{\varepsilon,t-1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Panel (a) reports moments of the time-series of \( \sigma_{\varepsilon,t} \) in the data and in various papers. Panel (b) reports correlations of \( \sigma_{\varepsilon,t} \) with macroeconomic variables in the data and in the models. Panel (c) reports the results of forecasting regressions in the data and in the models. RUBC is Bloom et al. (2018); CMR is Christiano, Motto, and Rostagno (2016). All variables are at the quarterly frequency.
A.1 Data calculations

A.1.1 Constructing implied volatility

For the Optionmetrics sample, we obtain at-the-money implied volatilities as the delta=50 IVs with maturity of 30 days from the Optionmetrics surface file.

For the BODB, the steps are as follows:

1. We calculate closing bid and ask prices for each option as the average of the final value and any other values recorded in the last 15 minutes of trading.

2. For each date/maturity/ticker combination, we take the strike immediately above and below the underlying price, as long as it is within 20 percent of the underlying.

3. Option prices are calculated as the midpoint between the bid and ask.

4. We drop all options with maturity less than 7 days.

5. The BODB reports a spot price. We replace the spot price with the value implied by put-call parity with a dividend of zero if the put-call parity implied price differs from the reported spot by more than 20 percent (this is to eliminate some clear data errors).

6. Implied volatilities are constructed using the Black–Scholes formula for European options ignoring dividends. For the one-month maturity, early exercise has generally very small effects on prices. We experimented by using the same method on data from Optionmetrics and comparing it to the implied volatilities that they report (which use a model for dividends and also account for early exercise) and we found the differences were quantitatively small.

7. We interpolate between maturities – and extrapolate where necessary – to get 30-day implied volatilities. Firm-level implied volatilities are set to have a maximum of 200 percent annualized and a minimum of zero (the interpolated values are winsorized).

8. The implied volatilities are then collapsed across firms weighting by market capitalization. We matched the tickers in the BODB to CRSP permco numbers to get market capitalization. In the large majority of cases, the BODB tickers are the same as the stock exchange tickers (they differ most for NASDAQ listings; the BODB manual, available online or on request from us, discusses this issue). The remainder are matched by hand where possible.

A.1.2 Calculating returns

For Optionmetrics, we directly use the data on closing bid and ask prices for options. We use the set of firms that was ever in the top 200 sorted by size during the Optionmetrics sample (1996–2019). For the BODB, the closing bid and ask are constructed as discussed
above and we use the entire available sample (which is tilted towards large firms). From there, the construction of returns is the same.

1. We drop all observations where the bid/ask spread is greater than 20 percent. We also apply step 5 above to the BODB data.

2. The two-week return is calculated by looking forward 10 trading days.

3. Return observations are dropped if at initiation the bid or ask price is less than 10 cents, the bid or ask volume is zero, or the maturity is less than 21 calendar days (the price filters are applied only at initiation so as not to introduce look-ahead bias).

4. We take the straddle with strike immediately above and below the spot, as long as they are within 20 percent of the spot and interpolate by log strike to construct an approximately at-the-money straddle. This requires having a valid straddle both above and below the spot, which often is not available.

5. Returns are collapsed across firms weighting by market capitalization at initiation. Returns are then interpolated and extrapolated to monthly maturities. For the one-month maturity, there must be a straddle available with a maturity of less than 60 days, and for the five-month maturity there must be available a straddle with maturity of at least 120 days (that is, we do not extrapolate by more than 30 days).

The S&P 500 straddle returns are constructed in a similar manner on data from the CME for the S&P 500 futures options and on Optionmetrics for SPX index options. In the BODB sample, the CME option returns are used to represent the market return, while the CBOE SPX options are used in with the Optionmetrics data.

A.2 How much of the variation is common?

For most of the analysis, we follow the literature in studying the common component in cross-sectional uncertainty. It is worth asking, though, how much of the variation in firm-level uncertainty is driven by that common component. To do so, we use the law of total variance,

\[
\text{var} (x_{i,t}) = E \left[ \text{var} (x_{i,t}) \right] + \text{var} [E_t (x_{i,t})] \tag{A.1}
\]

where \( \text{var}_t \) and \( E_t \) refer to the cross-sectional variance and average on date \( t \). The first term represents the residual variance after accounting for the cross-sectional average in each period, while the second term is the variance coming from that average. So the ratio of \( \text{var} [E_t (x_{i,t})] \) to the total variance represents the fraction of the total variance explained by
the cross-sectional mean in each period.

The variance decomposition identity (A.1) also holds with weights, so we weight by
market capitalization as above (normalizing the sum of market capitalization to 1 on each
date to give them equal weight overall). For $x_{i,t}$, we use total firm implied volatility measured
here as

$$
\sigma^2_{R,\varepsilon,i,t} \equiv \sigma^2_{i,t} - \beta^2_{i,R,t} \sigma^2_{mkt,t}
$$

where $\sigma^2_{R,\varepsilon,i,t}$ is a rolling beta estimated using the previous 12 months of daily data. When
we are just calculating the average of cross-sectional uncertainty across firms, the errors from
setting $\beta_i \approx 1$ somewhat cancel out across firms. Here, though, those errors will affect the
variance decomposition, so it is important to also examine what happens when we actually
estimate $\beta_i$.

### Variance decomposition for uncertainty measures

<table>
<thead>
<tr>
<th>Fraction from common component</th>
<th>Firm level</th>
<th>Sector level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total firm uncertainty ($\sigma^2_{i,t}$)</td>
<td>0.50</td>
<td>0.75</td>
</tr>
<tr>
<td>Firm-specific uncertainty ($\sigma^2_{R,\varepsilon,i,t}$)</td>
<td>0.40</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Depending on the measure, between 40 and 50 percent of the total variation in uncer-
tainty is due to a common component (measured as the cross-sectional average), which is
quantitatively consistent with the results in Herskovic et al. (2016). The fraction explained
by a common component is greater for total firm uncertainty, which is natural since that
includes market uncertainty, which affects all firms. The second column of the table above
reports similar results for measures of uncertainty averaged within two-digit sectors (i.e.
$\sum_{i \in S} w_{i,t} \sigma^2_{R,\varepsilon,i,t}$, where $S$ represents the set of firms in some sector). These results therefore
measure the extent to which cross-sectional uncertainty is similar across different sectors. In
this case, the fraction of the variation explained by the time-series component is 1.5 times
larger than in the firm-level case.

Overall, then, a surprisingly large amount of the total variation in cross-sectional or
idiosyncratic risk is driven by a common component that hits all parts of the economy,
which motivates us (and previous authors) to study a single common factor.
Figure A.1: Fraction of market capitalization covered by the options data

Note: The figure reports the ratio of total market capitalization for the firms for which we observe options data to the total market capitalization. Data before 1996 is from the Berkeley Options Dataset, and data after 1996 is from Optionmetrics. Shaded areas are NBER recessions.
Figure A.2: Average firm realized variance for different sets of firms

Note: The figure reports cross-sectional average realized variance for all firms in CRSP and for the firms for which we observe options. Option data before 1996 is from the Berkeley Options Dataset, and data after 1996 is from Optionmetrics. Shaded areas are NBER recessions.